Interleaved Direct Bandpass Sampling for Software Defined Radio/Radar Receivers

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February 24, 2016

Outline

- **Receiver Architectures**
- Bandpass Sampling Problem
- Interleaved ADC Sampling Model
- Complex Envelope Computation
- Computer Simulations
- Interleaved ADC Calibration
- Conclusions

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Receiver Architectures

- Consider modulated signal

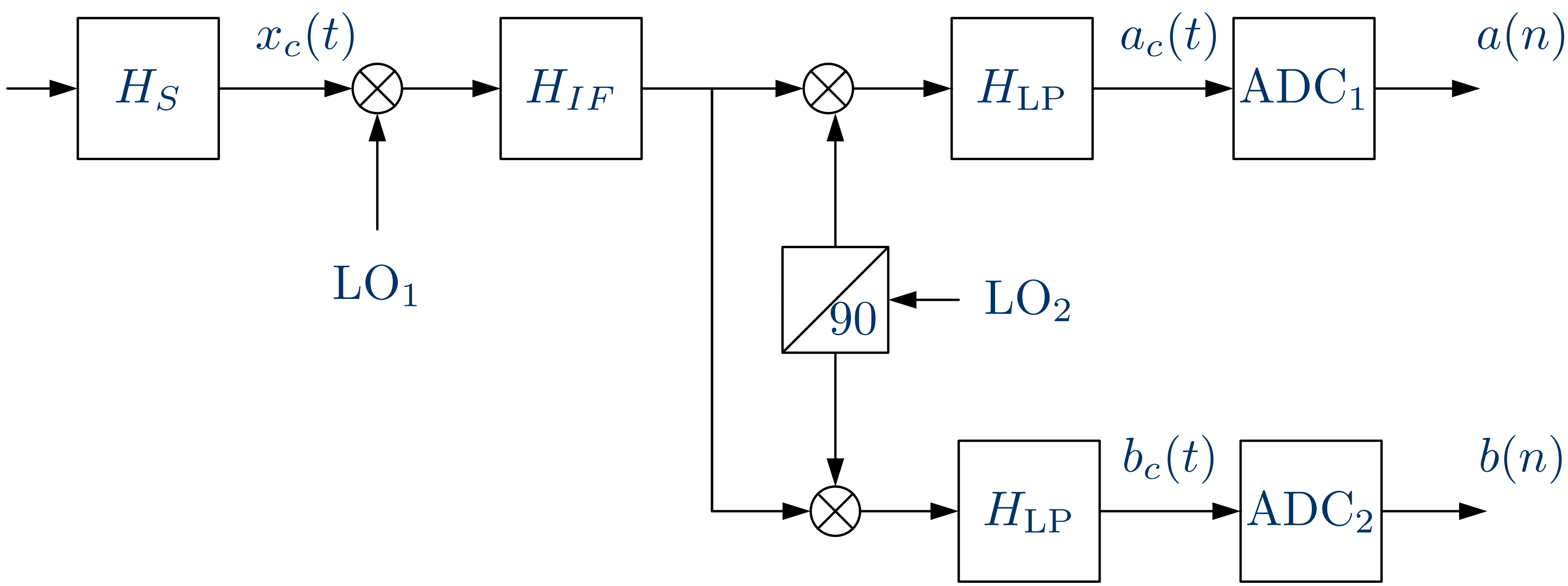
$$x_c(t) = \Re\{c_c(t)e^{j\Omega_c t}\}$$
$$= a_c(t)\cos(\Omega_c t) - b_c(t)\sin(\Omega_c t)$$

of $x_c(t)$, where Ω_c = carrier frequency and $c_c(t) = a_c(t) + jb_c(t) =$ complex envelope with bandwidth $B/2$.

- We seek to compute the sampled sequences $a(n) = a_c(nT_s')$ and $b(n) = b_c(nT_s')$ of the in phase and quadrature signal components, where $\Omega_s' > B$.

- Classical heterodyne and homodyne receiver architectures require analog filters and mixers.

Receiver Architectures (cont'd)



Heterodyne Receiver Architecture

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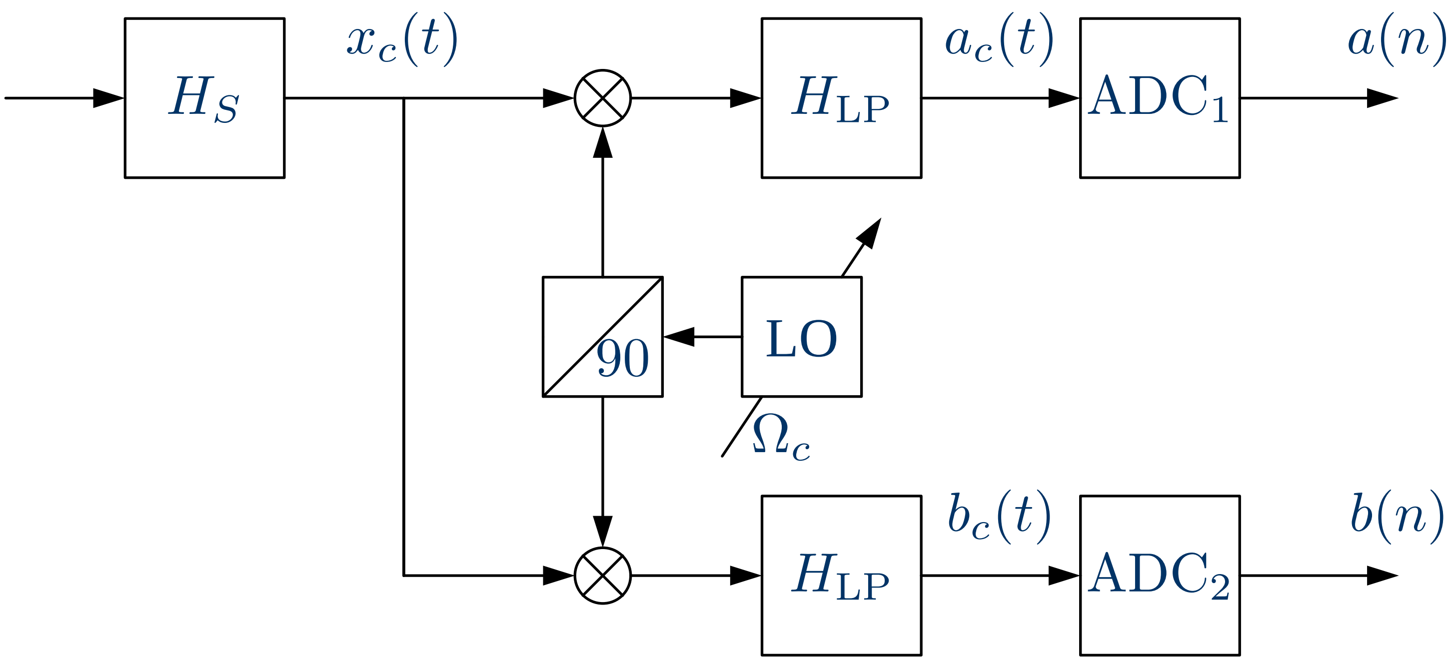
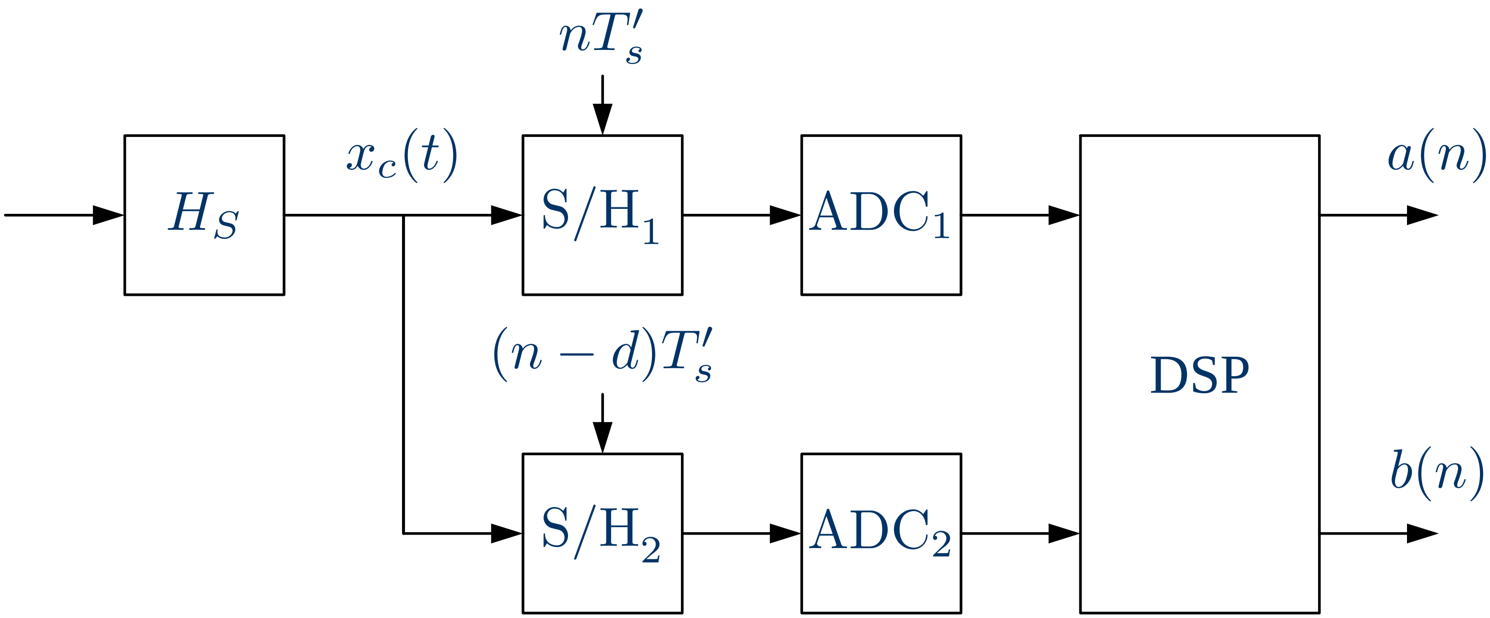
Receiver Architectures (cont'd)



Homodyne Receiver

Receiver Architectures (ct'd



Direct Interleaved Sampling Receiver

Uses a high quality tunable MEMS filter H_S, 2 wideband S/Hs (available from Inphi), but no mixer or phase shifter and fewer analog filters.

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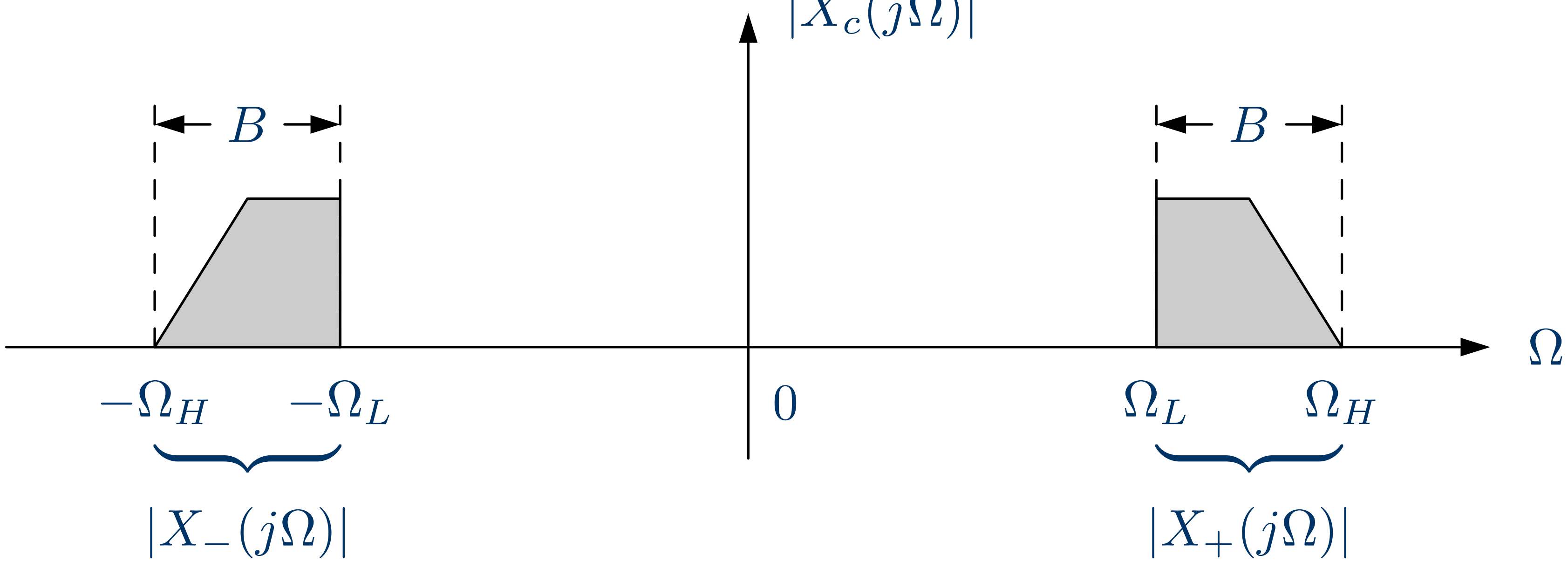
Bandpass Sampling Problem

- Consider a bandpass signal whose spectral support is $[-\Omega_H, -\Omega_L]$ and $[\Omega_L, \Omega_H]$, with occupied bandwidth $B = \Omega_H - \Omega_L$.



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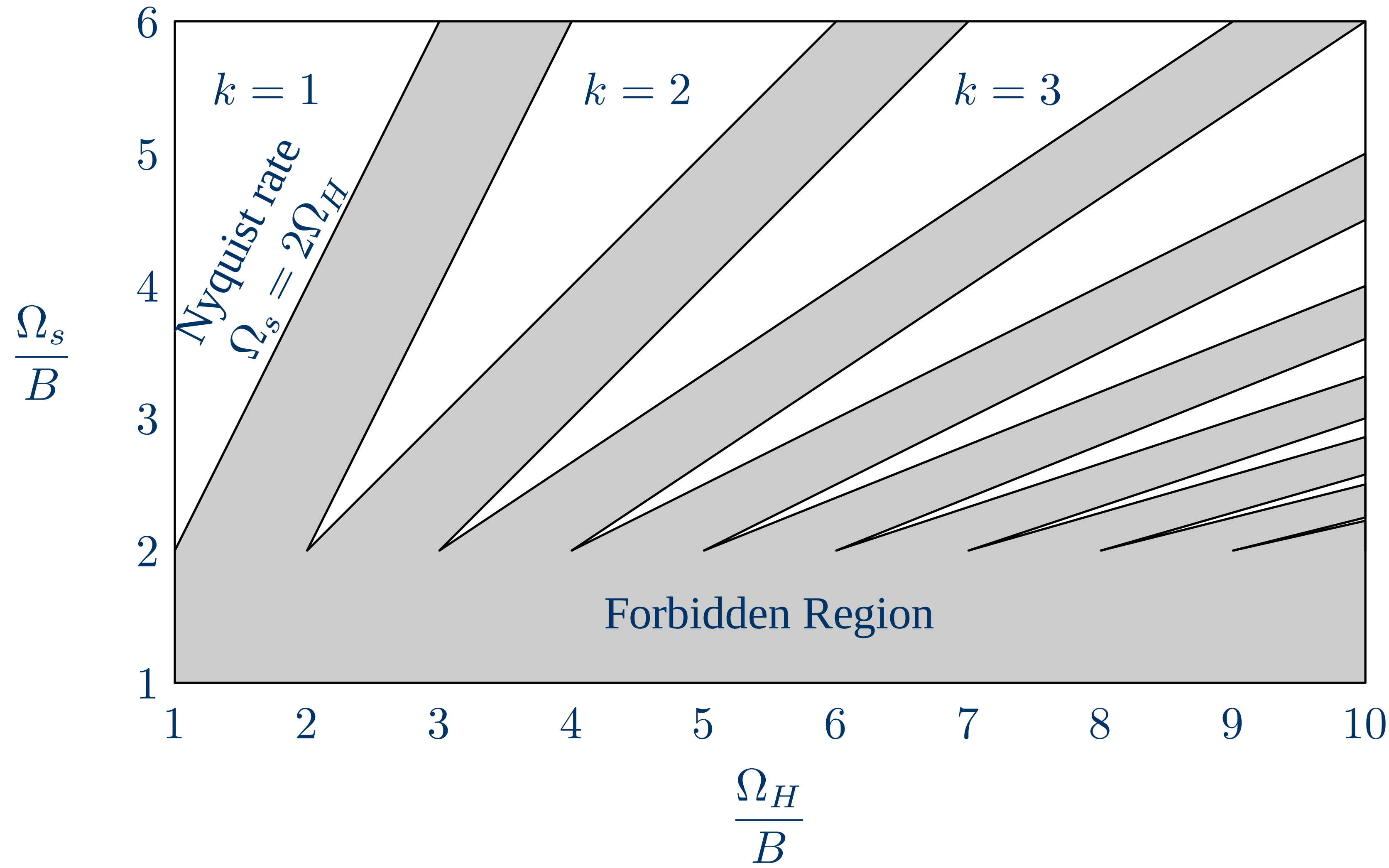
Bandpass Sampling Problem (cont'd)

- Due to overlap of copies $X_+(j(\Omega - k\Omega_s))$ and $X_-(j(\Omega - \ell\Omega_s))$ of the positive and negative components of the spectrum (with k, ℓ integer), it is not always possible to reconstruct $x_c(t)$ from its samples $x(n) = x_c(nT_s)$ if $\Omega_s > 2B$.
- Stronger conditions need to be satisfied. Let $k_M = \lfloor \Omega_H / B \rfloor$. Then Ω_s must satisfy

$$\frac{2\Omega_H}{kB} \leq \frac{\Omega_s}{B} \leq \frac{2\Omega_L}{(k-1)B}$$

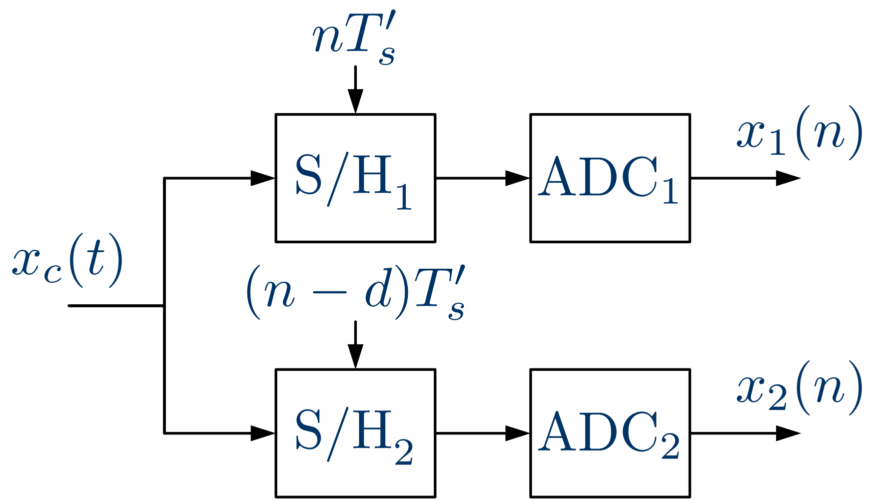
with k integer such that $1 \leq k \leq k_M$.

Bandpass sampling (cont'd)



Bandpass Sampling (cont'd)

- Kohlenberg (1953), Coulson (1995), Lin and Vadyanathan (1998) recognized that $x_c(t)$ can be recovered from the samples $x_1(n) = x_c(nT_s')$ and $x_2(n) = x_c((n-d)T_s')$ of two interleaved ADCs, as long as the overall sampling rate $\Omega_s = 2\Omega_s' > 2B$. But some offsets, such as $d = 1/2$, are forbidden.



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Interleaved ADC Sampling Model

- Consider the representation

$$x_c(t) = \Re\{c_c(t)e^{j\Omega_c t}\}$$

of $x_c(t)$, where $\Omega_c =$ carrier frequency and $c_c(t) = a_c(t) + jb_c(t) =$ complex envelope with bandwidth $B/2$.

- Unlike Kohlenberg and others who considered the problem of recovering $x_c(t)$ from its interleaved samples, we seek to evaluate the sampled complex envelope $c(n) = c_c(nT_s)$ from the interleaved samples, i.e. we jointly sample and demodulate.

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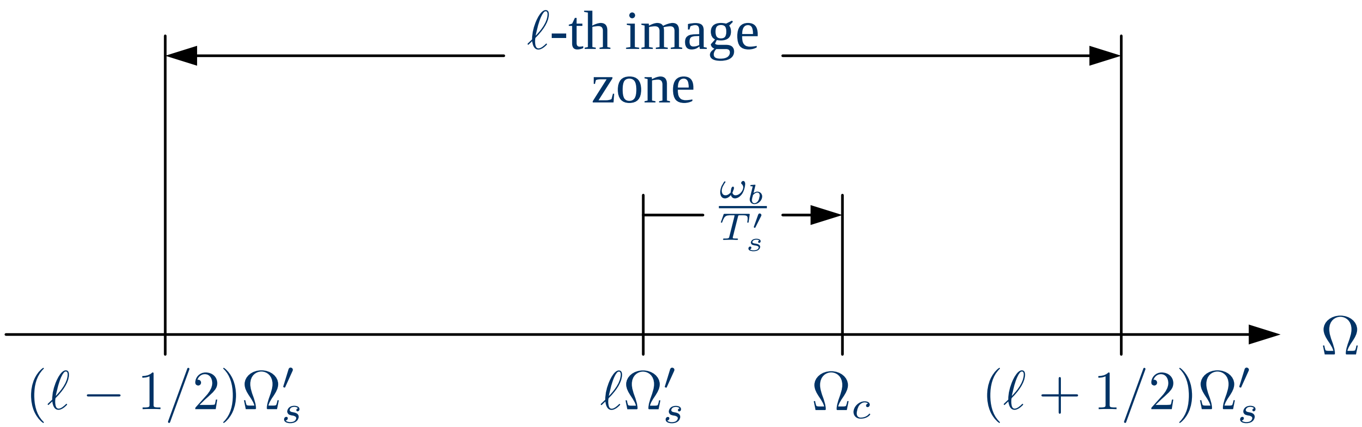
Interleaved ADC Sampling Model (cont'd)

- Let $\ell = \text{round}(\Omega_c/\Omega_s')$ and

$$\omega_b = \left(\frac{\Omega_c}{\Omega_s'} - \ell\right)2\pi \ ,$$

so that $-\pi \leq \omega_b < \pi$.



Interleaved ADC Sampling Model (cont'd)

With $\Omega'_s > B$, if $c(n) = c_c(nT'_s) =$ sampled envelope, we have

$$x_1(n) = \Re\{c(n)e^{j\Omega_c T'_s n}\} = \Re\{c(n)e^{j\omega_b n}\}$$

and

$$\begin{aligned} x_2(n) &= \Re\{(f * c)(n)e^{j\Omega_c T'_s(n-d)}\} \\ &= \Re\{(f * c)(n)e^{j\omega_b(n-d)}e^{-j2\pi\ell d}\} \end{aligned}$$

where $f(n) =$ impulse response of the fractional delay filter $F(e^{j\omega}) = e^{-j\omega d}$ for $-\pi < \omega \leq \pi$. Let

$$G(e^{j\omega}) = \begin{cases} e^{-j2\pi(\ell+1)} & -\pi \leq \omega < -\pi + \omega_b \\ e^{-j2\pi\ell d} & -\pi + \omega_b \leq \omega < \pi \end{cases}$$

for $\omega_b \geq 0$, and

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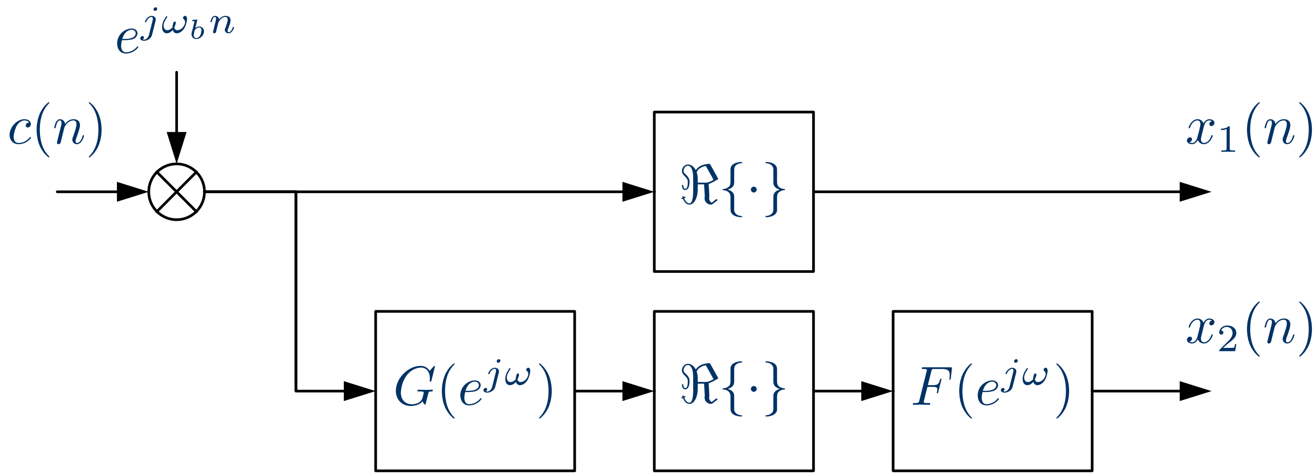
Interleaved ADC Sampling Model (cont'd)

$$G(e^{j\omega}) = \begin{cases} e^{-j2\pi\ell d} & -\pi \le \omega \le \pi + \omega_b \\ e^{-j2\pi(\ell-1)d} & \pi + \omega_b \le \omega < \pi \end{cases}$$

for $\omega_b < 0$. This gives the model shown below, where the goal is to recover $c(n)$ from $x_1(n)$ and $x_2(n)$.



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Complex Envelope Computation

We have

$$\begin{bmatrix} X_1(e^{j\omega}) \\ X_2(e^{j\omega}) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & F(e^{j\omega}) \end{bmatrix} \mathbf{M}(e^{j\omega}) \begin{bmatrix} C(e^{j(\omega-\omega_b)}) \\ C^*(e^{-j(\omega+\omega_b)}) \end{bmatrix}$$

where

$$\mathbf{M}(e^{j\omega}) = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ G(e^{j\omega}) & G^*(e^{-j\omega}) \end{bmatrix}$$

and $X_1(e^{j\omega})$, $X_2(e^{j\omega})$ and $C(e^{j\omega})$ = DTFTs of $x_1(n)$, $x_2(n)$ and $c(n)$. So $C(e^{j(\omega-\omega_b)})$ can be recovered from $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ as long as $\mathbf{M}(e^{j\omega})$ is invertible.

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Complex Envelope Computation (cont'd)

The determinant $D(e^{j\omega})$ of $\mathbf{M}(e^{j\omega})$ is given by

$$D(e^{j\omega}) = \frac{j}{2}\sin(2\pi\ell d)$$

for $0 \leq \omega \leq \pi - |\omega_b|$ and

$$D(e^{j\omega}) = \frac{j}{2}e^{j\pi \mathrm{sgn}(\omega)d}\sin(\pi(2\ell + \mathrm{sgn}(\omega_b))d\ ,$$

for $\pi - |\omega_b| \leq \omega < \pi$. Accordingly $D(e^{j\omega})$ is nonzero as long as $\ell \geq 1$ and the offset d is not equal to one of the forbidden values

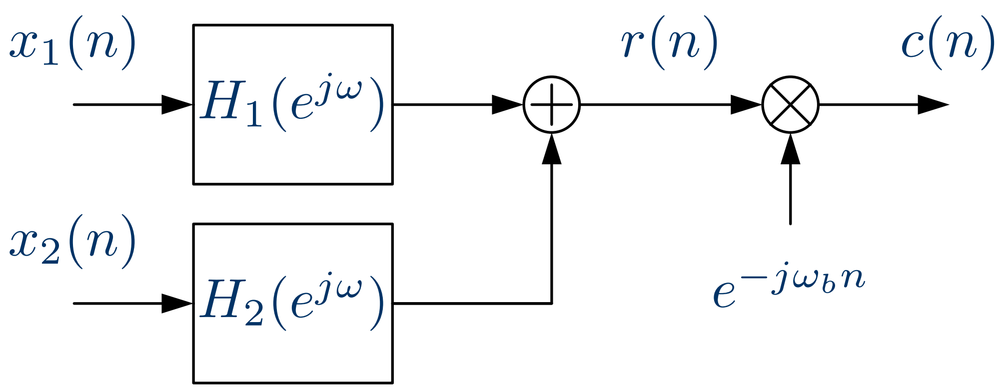
$$d_m^i = \frac{m}{2\ell}\ \ ,\ \ d_q^e = \frac{q}{2\ell + \mathrm{sgn}(\omega_b)}$$

with m and q integers such that $0 \leq m \leq 2\ell - 1$ and $1 \leq q \leq 2\ell + \mathrm{sgn}(\omega_b) - 1$.

Complex Envelope Computation (cont'd)

The complex envelope $c(n)$ can be computed as shown below, where

$$\begin{aligned} \begin{bmatrix} H_1(e^{j\omega}) & H_2(e^{j\omega}) \end{bmatrix} &= \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{M}^{-1}(e^{j\omega}) \begin{bmatrix} 1 & 0 \\ 0 & F^{-1}(e^{j\omega}) \end{bmatrix} \\ &= \frac{1}{2D(e^{j\omega})} \begin{bmatrix} G^*(e^{-j\omega}) - F^{-1}(e^{j\omega}) \end{bmatrix} . \end{aligned}$$



Complex Envelope Computation (cont'd)

- The filters $H_1(e^{j\omega})$ and $H_2(e^{j\omega})$ are noncausal IIR with impulse responses

$$\Re\{h_1(n)\} = \delta(n) \;,\; \Re\{h_2(n)\} = 0$$

$$\begin{aligned}\Im\{h_1(n)\} = & -\cot(\pi(2\ell + \mathrm{sgn}(\omega_b))d)\delta(n) \\ & + \Big(\cot(\pi(2\ell + \mathrm{sgn}(\omega_b))d) - \cot(2\pi\ell d)\Big)\frac{\sin((\pi - |\omega_b|)n)}{\pi n}\end{aligned}$$

$$\begin{aligned}\Im\{h_2(n)\} = & \frac{\sin((\pi - |\omega_b|)(n+d))}{\pi\sin(2\pi\ell d)(n+d)} \\ & - \frac{\sin((\pi - |\omega_b|)(n+d) - \pi d)}{\pi\sin(\pi(2\ell + \mathrm{sgn}(\omega_b))d)(n+d)} \;.\end{aligned}$$

- They can be approximated by causal FIR filters of order M=2L by shifting the responses by L and applying a Kaiser window of order M.

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Complex Envelope Computation (cont'd)

- The quadrature sampling scheme proposed by Brown (1979) requires $\Omega_s' = \Omega_c/\ell$ ($\omega_b = 0$) and $d = \frac{1}{4\ell} + \frac{q}{\ell}$ with q integer. In this case

$$x_1(n) = a_c(nT_s') \ , \ \ x_2(n) = b_c((n-d)T_s')$$

  are the sampled in-phase and quadrature components of the complex envelope

- Impractical for software defined radio since it ties the sampling frequency to the carrier frequency.

- The proposed scheme requires only $\Omega_s > 2B$ and $\ell \geq 1$. It is well adapted to software defined radio: if Ω_c changes, ℓ and ω_b change and the filters $h_1(n)$ and $h_2(n)$ can be recomputed by windowing their closed form expressions.

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Computer Simulations

- The bandpass signal $x_c(t)$ has envelope

$$c_c(t) = 2\cos(400 \times 10^6 \times 2\pi t) \\ +j[\sin(400 \times 10^6 \times 2\pi t) + \cos(175 \times 10^6 \times 2\pi t)]$$

  with bandwith $(B/2)/2\pi = 400\text{MHz}$, and carrier frequency $F_c = \Omega_c/(2\pi) = 5.15\text{GHz}$.

- The sub-ADC sampling frequency $F_s' = \Omega_s'/(2\pi) = 1\text{GHz}$ is above $B = 800\text{MHz}$.

- $F_c = 5F_s' + 150$, so $\ell = 5$ and $\omega_b = 0.3\pi$.

- The sampled envelope

$$c(n) = \frac{3}{2}e^{j0.8\pi n} + \frac{1}{2}e^{-j0.8\pi n} + \frac{j}{2}[e^{j0.35\pi n} + e^{-j0.35\pi n}]$$

  has tones at $\pm 0.8\pi$ and $\pm 0.35\pi$.

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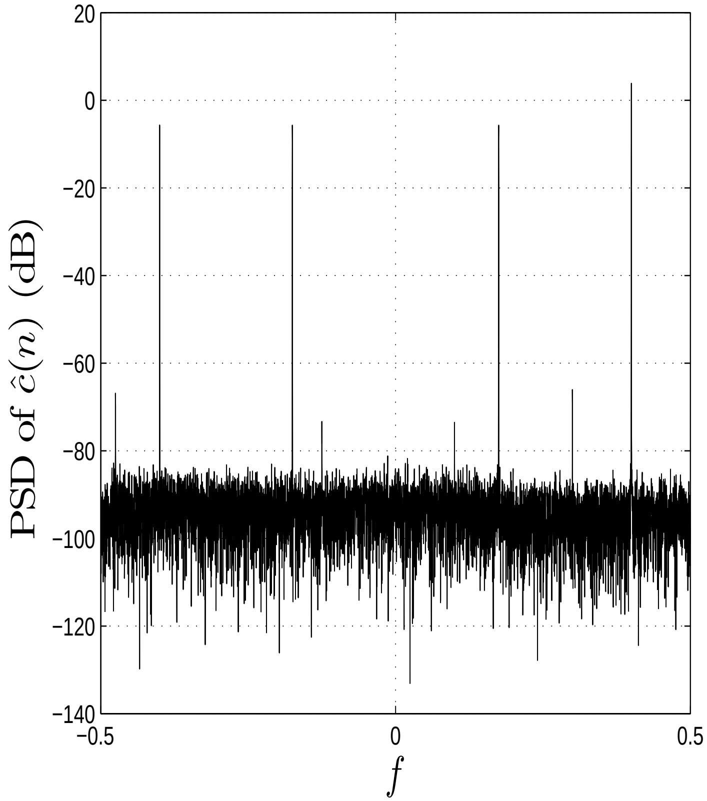
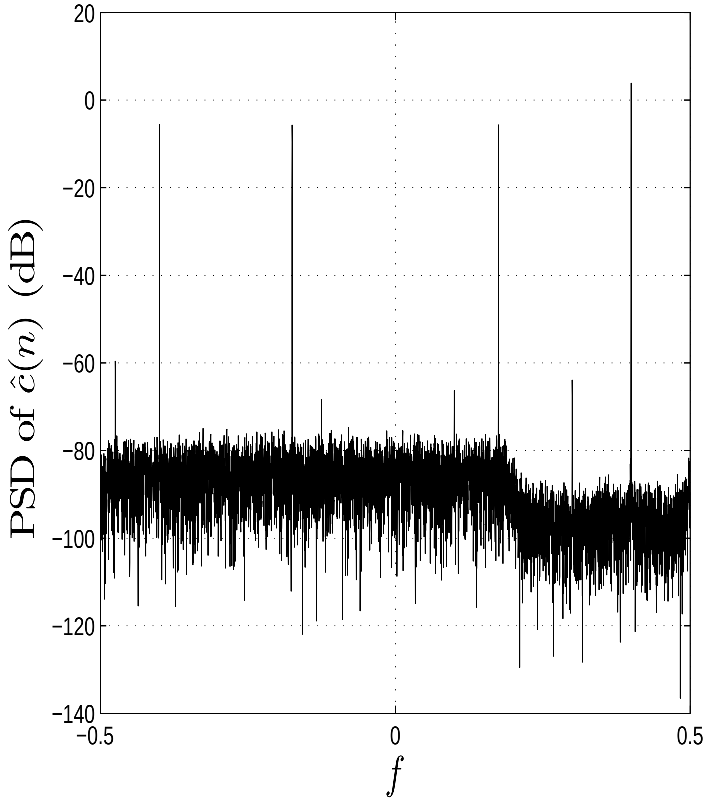
Computer Simulations (cont'd)



- Timing offset $d = 0.425$, between $d_4^i = 0.4$ and $d_5^e = 0.454$.
- Additive noises with SNR =62dB added to $x_1(n)$ and $x_2(n)$ to model thermal, quantization noises
- Kaiser windows of order $M = 60$ and $\beta = 6$ used to approximate filters H_1 and H_2.
- MSE= -53.54dB, SFDR = 65dB.
- Secondary tones = residual components of $e^{-j2\omega_b n} c^*(n)$.

Computer Simulations (cont'd)



- Timing offset $d = d_4^i + 0.001$ close to a forbidden offset.
- $D(e^{j\omega})$ almost singular over $[-0.7\pi, 0.7\pi]$, so filter gains inaccurate over this band.
- Translated to $[-\pi, 0.4\pi]$ by $e^{-j0.3\pi n}$ demodulation.
- MSE = -47.27dB, SFDR =55dB.

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Interleaved ADC Calibration

- The effect of timing-skew mismatches $\delta = d - d_0$ with $d =$ true mismatch, $d_0 =$ nominal mismatch, grows as increases. Due to the $2\pi\ell d$ phase shift of $G(e^{j\omega})$.
- ADC_1 and ADC_2 cannot share the sample S/H since $d \neq 1/2 \Rightarrow$ mismatches cannot be avoided.
- TIADC calibration is therefore needed. We propose a blind calibration method under the assumption that the signal is oversampled with oversampling ratio $\alpha = 1 - 2B/\Omega_s$.

Interleaved ADC Calibration

- If $\gamma = g - 1 =$ relative gain mismatch, if

$$\boldsymbol{\theta} = \begin{bmatrix} g \\ d \end{bmatrix} \ , \ \boldsymbol{\theta}_0 = \begin{bmatrix} 1 \\ d_0 \end{bmatrix}$$

denote the true and nominal parameter vectors ($g =$ relative channel gain), for small mismatches we use first-order expansions

$$\begin{aligned} H_1(e^{j\omega}, d) &= H_{10}(e^{j\omega}) + \delta H_{11}(e^{j\omega}) \\ H_2(e^{j\omega}, \boldsymbol{\theta}) &= (1-\gamma)[H_{10}(e^{j\omega}) + \delta H_{11}(e^{j\omega})] \,. \end{aligned}$$

- Due to oversampling, $C(e^{j\omega}) = 0$ over $J = [(1-\alpha)\pi, (1+\alpha)\pi]$ $\bmod (2\pi)$, so $r(n, \boldsymbol{\theta}) = c(n) e^{j\omega_b n}$ has no power in the band $J + \omega_b$ $\bmod (2\pi)$.

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Interleaved ADC Calibration

- let $h_{\mathrm{BP}}(n)$ = impulse response of bandpass filter

$$H_{\mathrm{BP}}(e^{j\omega}) = \begin{cases} 1 & \omega \in I \\ 0 & \text{otherwise} . \end{cases}$$

  Then the error signal

$$e(n, \hat{\boldsymbol{\theta}}) = h_{\mathrm{BP}}(n) * r(n, \hat{\boldsymbol{\theta}})$$

  is zero for $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}$, nonzero otherwise.

- The blind estimation algorithm minimizes adaptively

$$J(\hat{\boldsymbol{\theta}}) = E[|e(n, \hat{\boldsymbol{\theta}}|^2]$$

  by using the stochastic gradient scheme

$$\hat{\boldsymbol{\theta}}(n+1) = \hat{\boldsymbol{\theta}}(n) - \mu \Re\{e^*(n, \hat{\boldsymbol{\theta}}(n) \nabla_{\hat{\boldsymbol{\theta}}} e(n, \hat{\boldsymbol{\theta}}(n))\} .$$

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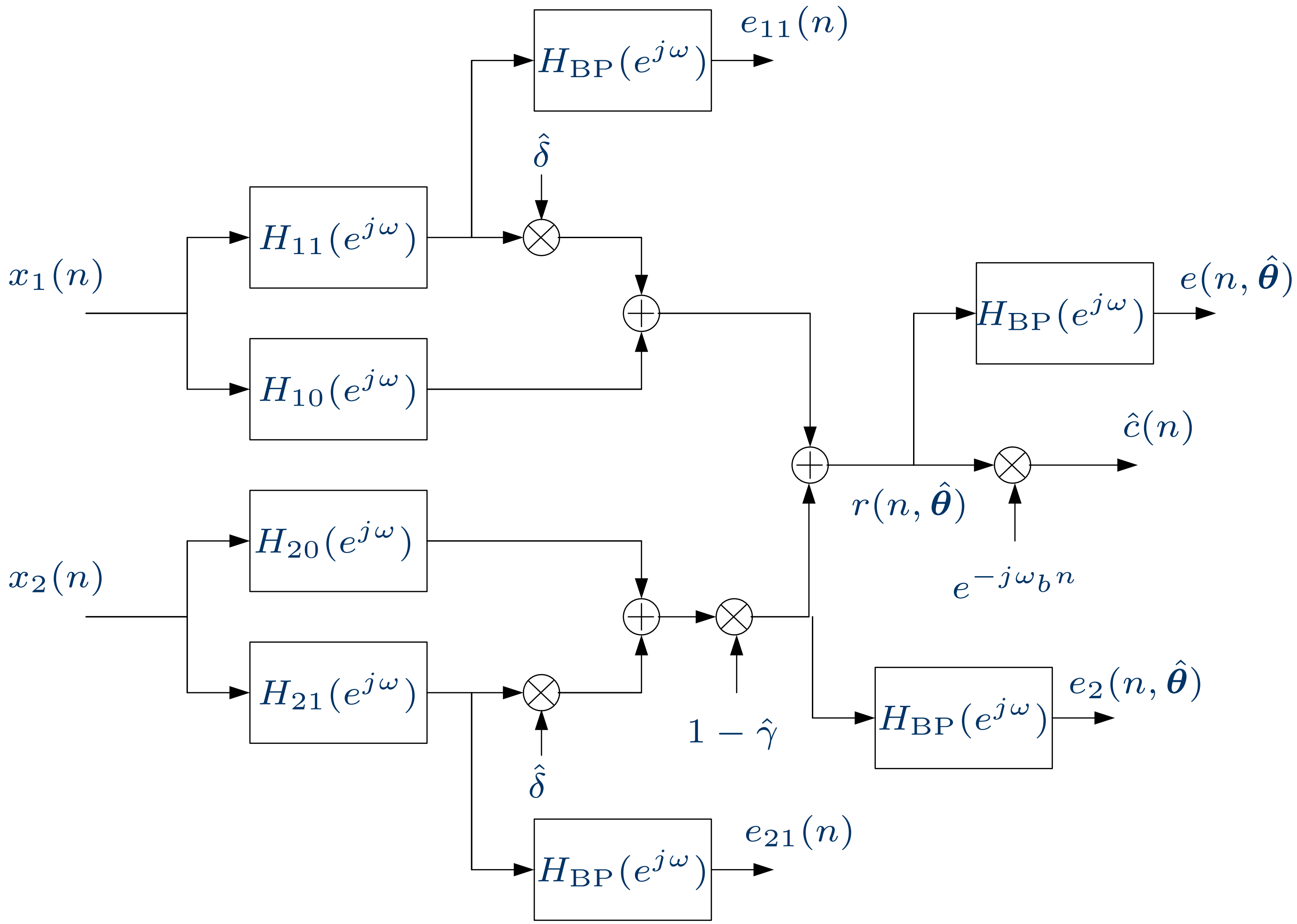
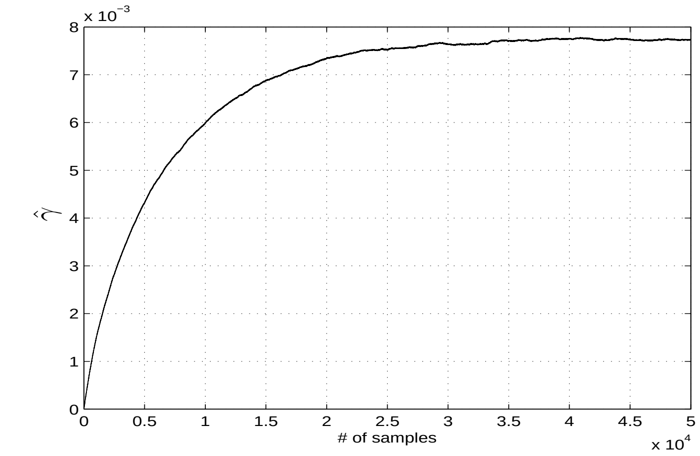
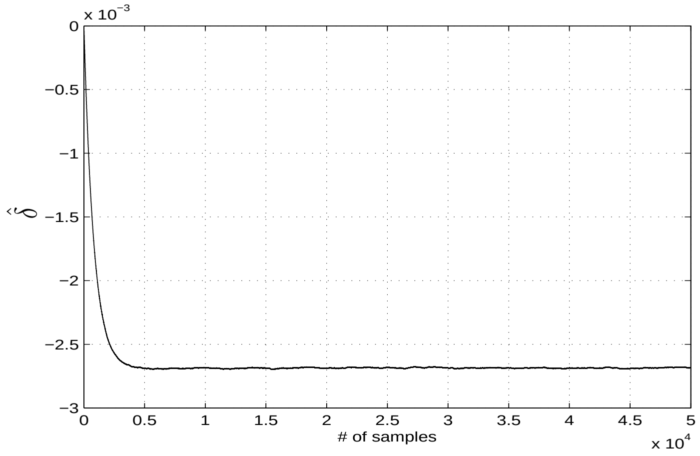
Interleaved ADC Calibration (cont'd)



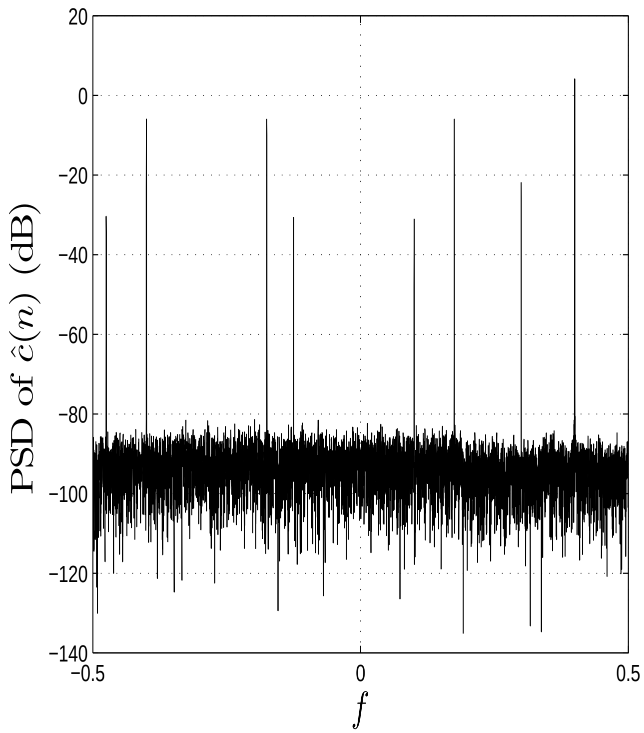
Interleaved ADC Calibration (cont'd)

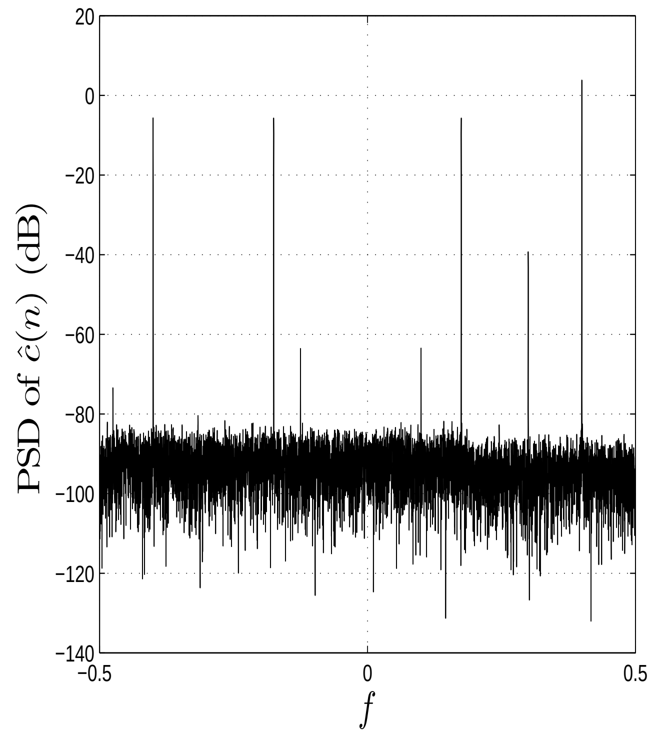




- Mismatches: $\gamma = 10^{-2}$, $\delta = -0.25 \times 10^{-2}$.
- $I = [-0.9\pi, -0.5\pi]$, FIR approximation of H_{BP} with Kaiser window of order $M = 80$.
- Step sizes $\mu_\gamma = 10^{-3}$, $\mu_\delta = 10^{-5}$.
- $L = 5 \times 10^4$ samples. Final estimates $\hat{\gamma}(L) = 0.77 \times 10^{-2}$, $\hat{\delta} = -0.27 \times 10^{-2}$.

Interleaved ADC Calibration (cont'd)





Before calibration: MSE=-17.8dB, SFDR=25dB

After calibration: MSE=-36.51dB, SFDR=43dB.

Interleaved ADC Calibration (cont'd)

Carrier Freq. ℓ	2.15GHz 2	3.15Gz 3	4.15GHz 4	5.15 GHz 5
MSE, Before (dB) MSE, After (dB)	-25.51 -50.89	-22.24 -46.01	-19.76 -40.97	-17.79 -36.52
$\hat{\gamma}(L)(\times 10^{-3})$ $\hat{\delta}(L)\ (\times 10^{-3})$	10 2.5	9.7 2.6	8.8 2.6	7.7 2.7

The performance of the envelope reconstruction scheme before and after calibration degrades as ℓ increases. Reason: the first order expansion of correction filters H_1 and H_2 becomes inaccurate. As soon as g and d are estimated, the exact filters H_1 and H_2 should be used instead of their first order approximations.

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Conclusions

- A direct complex envelope sampling scheme for bandpass signals with nonuniformly interleaved ADCs has been proposed. In combination with tunable RF band selection filters and high bandwidth S/H circuits, provides an approach to software defined radio/radar.
- TIADC calibration is required due to the sensitivity of the sampling system to timing skew mismatches.
- The proposed sampling and blind calibration system work well for small to moderate values of ℓ (Ω_c/B). The calibration can be improved for large values of ℓ by iterating the calibration using the most recent $\boldsymbol{\theta}$ estimate as reference in the first-order expansions of $H_1(e^{j\omega}, d)$ and $H_2(e^{j\omega}, \boldsymbol{\theta})$.

Thank you!