Modeling Transformer Winding Behavior of Multi-Output Power Supplies using Mutual Impedance Effects

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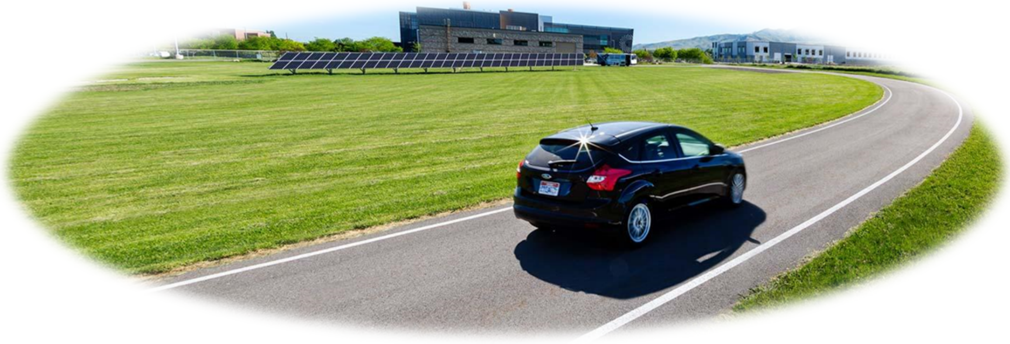
IEEE MAGNETICS



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Overview

- This presentation introduces a circuit simulation model that includes both inductive coupling and mutual resistance effects
- Mutual inductance and mutual resistance concepts are reviewed
- Modeling approach particularly suited for cases not covered by Dowell’s method such as transformers with multiple output windings
- Model parameters are extracted in the frequency domain, but the models also work well in the time domain
- A transformer for a Phase-Shifted Bridge is used as an example to illustrate the modeling approach

Dowell's Method Limitations

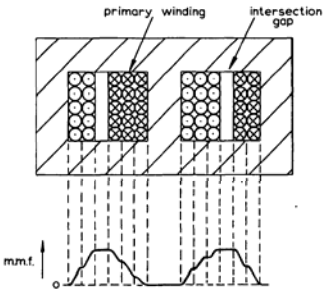


Fig. 2
A typical transformer, with associated m.m.f. diagram
○ ⊗ Indicates current directions

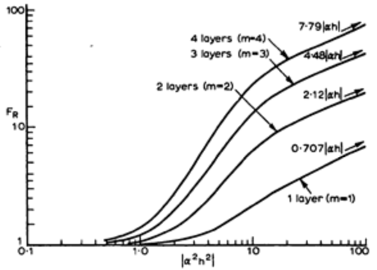
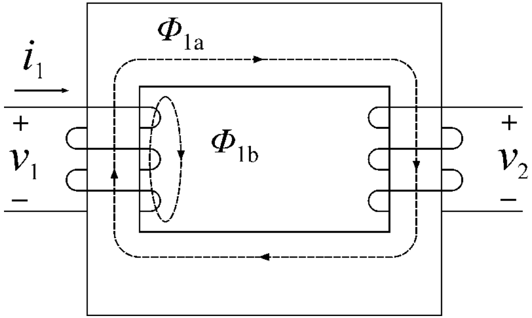


Fig. 8
F_R as a function of $|\alpha^2h^2|$ for winding portions with integral numbers of layers

[1]

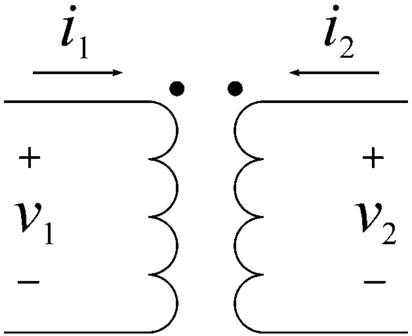
- Used to calculate the ac resistance of transformer windings
- Assumes infinite magnetizing inductance (equal and opposite amp-turns)
  – not intended for low permeability or gapped cores (amp-turns unequal and possible fringing loss)
- Assumes one independent current variable (currents scaled by turns ratios)
  – Interleaved windings are supported if they are connected in series
  – Multiple outputs with independent load currents not supported
  – Windings connected in parallel not supported because the current sharing ratio is unknown

Magnetic Coupling Review



- Two windings are coupled when some of the magnetic flux produced by currents flowing in either of the windings passes through both windings
- Only part of the flux produced by a current in one winding reaches other windings
- Flux which doesn't pass through both windings is called leakage flux
- Magnetic coupling can also be modeled in terms of self and mutual impedances

Self and Mutual Impedance Equations [2-5]



$$v_1 = Z_{11} i_1 + Z_{12} i_2$$

$$v_2 = Z_{21} i_1 + Z_{22} i_2$$

$$v_1 = (R_{11} + j\omega L_{11}) i_1 + (R_{12} + j\omega L_{12}) i_2$$

$$v_2 = (R_{21} + j\omega L_{21}) i_1 + (R_{22} + j\omega L_{22}) i_2$$

$R_{12} = R_{21} =$ Mutual Resistance

$L_{12} = L_{21} =$ Mutual Inductance

Impedance Matrix Equation for N Windings

$$\begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_N \end{bmatrix}$$

- A set of coupled windings can be modeled with a matrix equation that relates frequency-domain winding voltages and currents with an impedance matrix [6]
- The values of impedance matrix elements can be obtained through FEA simulations or extracted from measurements [7]
- The impedance matrix values vary with frequency
- Impedance matrices are symmetric

ANSYS Maxwell Impedance Matrix Results

DesignVariation : Ipeak1='1A' Ipeak2='-3A' Ipeak3='-3A' Ipeak4='1A'
Solution : Setup1 : LastAdaptive
Parameter : ReduceMatrix
Inductance Unit: nH

Adaptive Freq : 1Hz

R,L

	Winding1	Winding2	Winding3	Winding4
Winding1	0.0090854, 1.942E+05	3.1879E-09, 64607	3.1406E-09, 64449	9.3019E-09, 1.9268E+05
Winding2	3.1879E-09, 64607	0.0015571, 21581	1.0364E-09, 21535	3.0759E-09, 64376
Winding3	3.1406E-09, 64449	1.0364E-09, 21535	0.0017664, 21575	3.0606E-09, 64519
Winding4	9.3019E-09, 1.9268E+05	3.0759E-09, 64376	3.0606E-09, 64519	0.01418, 1.9399E+05

10000Hz

R,L

	Winding1	Winding2	Winding3	Winding4
Winding1	0.035178, 1.9285E+05	0.0086129, 64162	0.0083757, 64014	0.024021, 1.9141E+05
Winding2	0.0086129, 64162	0.0044826, 21433	0.0028806, 21389	0.008286, 63949
Winding3	0.0083757, 64014	0.0028806, 21389	0.0046813, 21429	0.008507, 64089
Winding4	0.024021, 1.9141E+05	0.008286, 63949	0.008507, 64089	0.039969, 1.927E+05

15848.9319246111Hz

R,L

	Winding1	Winding2	Winding3	Winding4
Winding1	0.046427, 1.9273E+05	0.0123, 64123	0.011947, 63976	0.034285, 1.913E+05
Winding2	0.0123, 64123	0.005745, 21419	0.0041158, 21376	0.011827, 63911
Winding3	0.011947, 63976	0.0041158, 21376	0.0059462, 21416	0.012177, 64050
Winding4	0.034285, 1.913E+05	0.011827, 63911	0.012177, 64050	0.051245, 1.9258E+05

25118.8643150958Hz

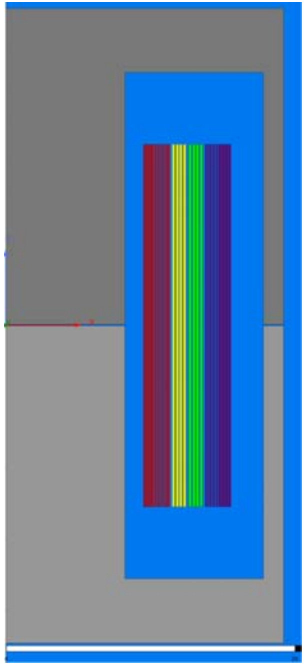
R,L

	Winding1	Winding2	Winding3	Winding4
Winding1	0.062141, 1.9263E+05	0.017424, 64088	0.01687, 63942	0.0484, 1.9121E+05
Winding2	0.017424, 64088	0.0075022, 21408	0.0058219, 21364	0.016704, 63877
Winding3	0.01687, 63942	0.0058219, 21364	0.0077053, 21404	0.017271, 64016
Winding4	0.0484, 1.9121E+05	0.016704, 63877	0.017271, 64016	0.066999, 1.9248E+05

Transformer Comparison

Two transformers with different insulation thickness between the windings

3 layers 2 mil Nomex



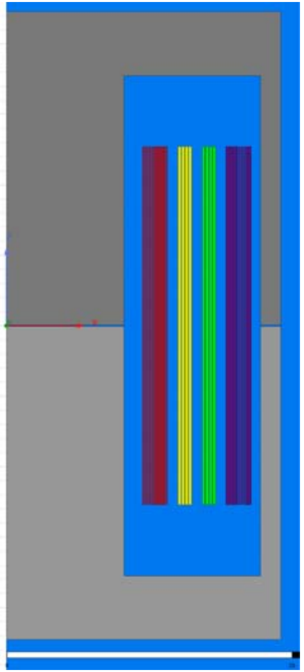
Measured Capacitance
between windings

1-2: 122 pF
2-3: 151 pF
3-4: 174 pF

SRF: 1.03 MHz

Measured Inductances @10 kHz
Winding 1: 181 μH
Winding 2: 20.2 μH
Winding 3: 20.2 μH
Winding 4: 181 μH

10 layers 2-mil Nomex



Measured Capacitance
between windings

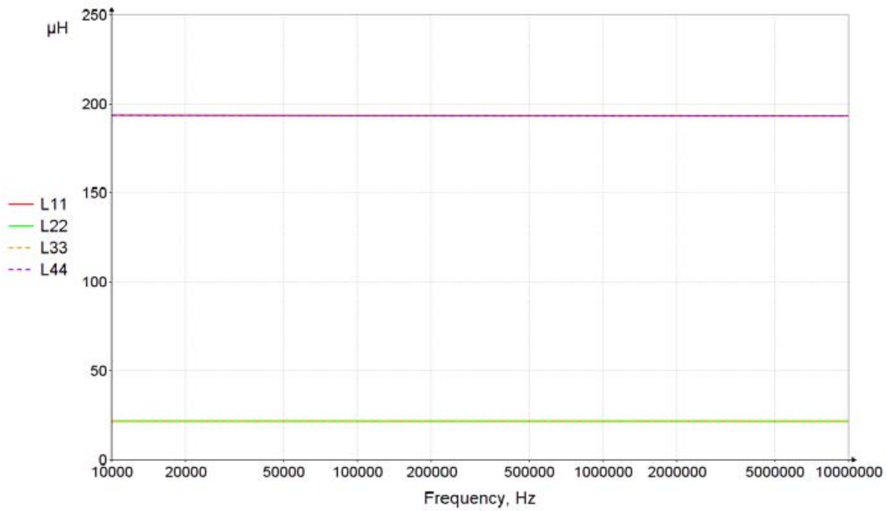
1-2: 55 pF
2-3: 61 pF
3-4: 65 pF

SRF: 998 kHz

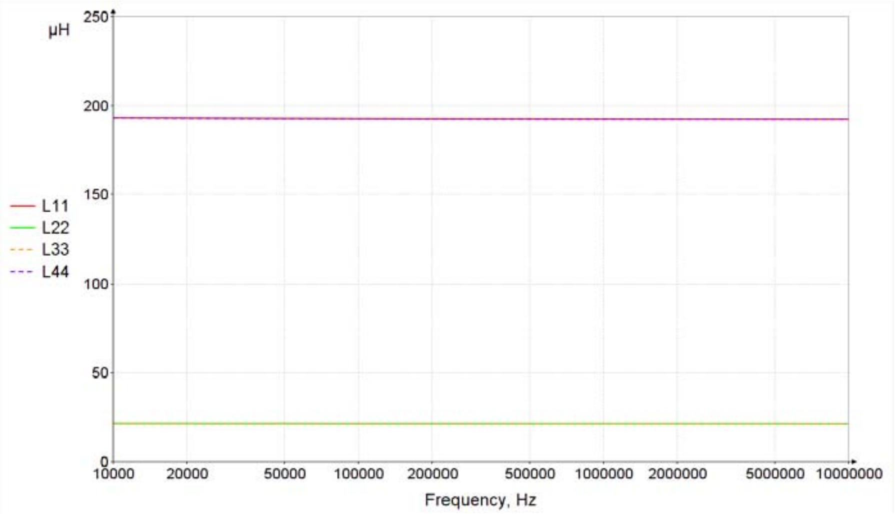
Measured Inductances@ 10 kHz
Winding 1: 192 μH
Winding 2: 21.4 μH
Winding 3: 21.4 μH
Winding 4: 192 μH

FEA Self Inductances

3 layers 2 mil Nomex

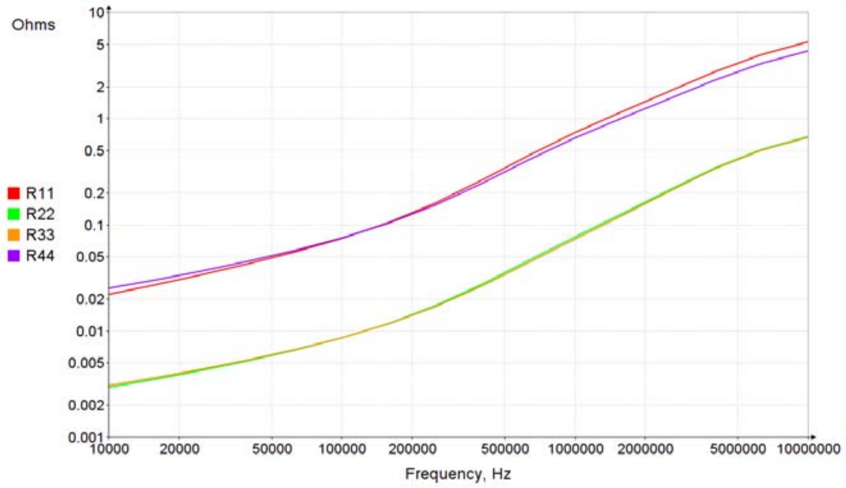


10 layers 2 mil Nomex

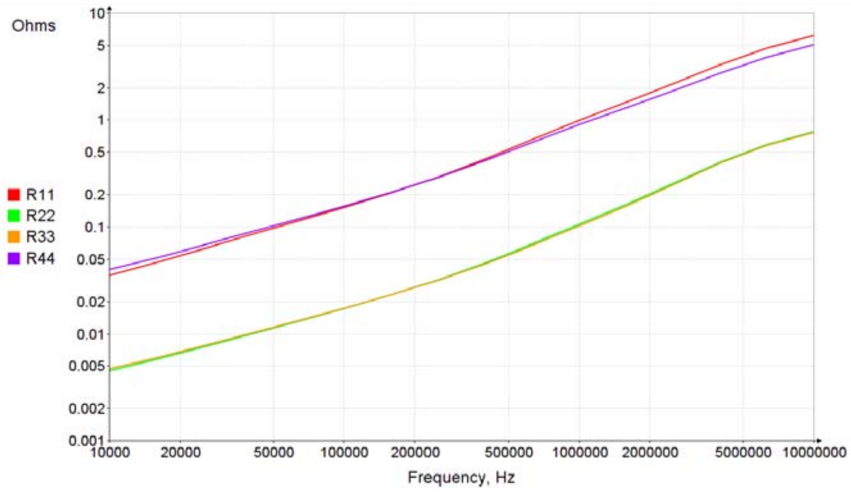


Self Resistances

3 layers 2 mil Nomex

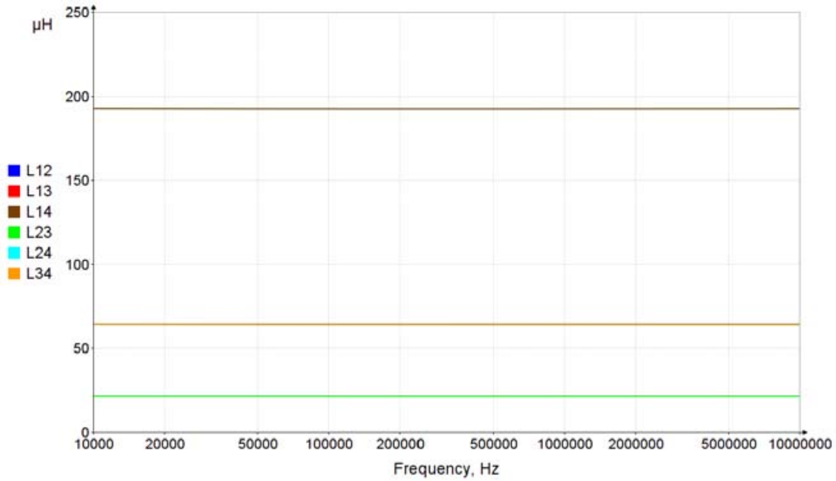


10 layers 2 mil Nomex

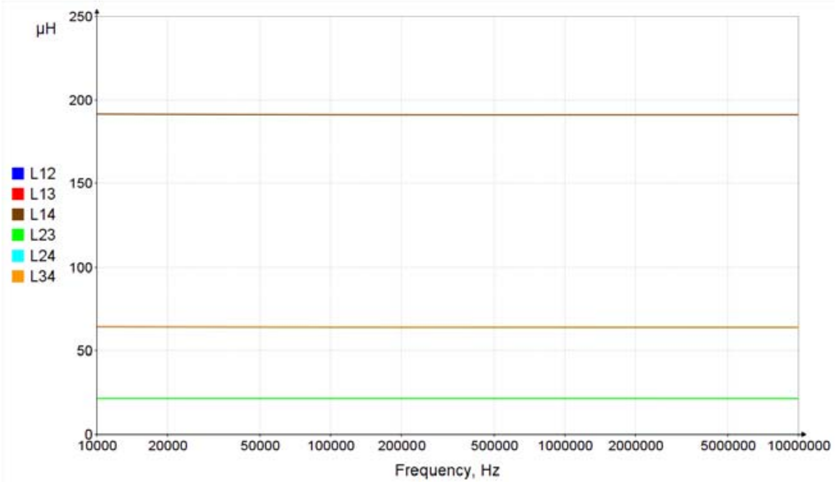


Mutual Inductances

3 layers 2 mil Nomex



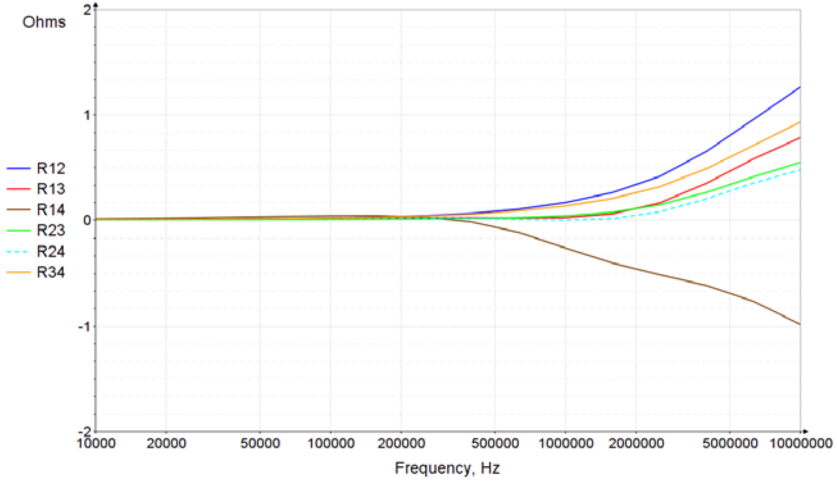
10 layers 2 mil Nomex



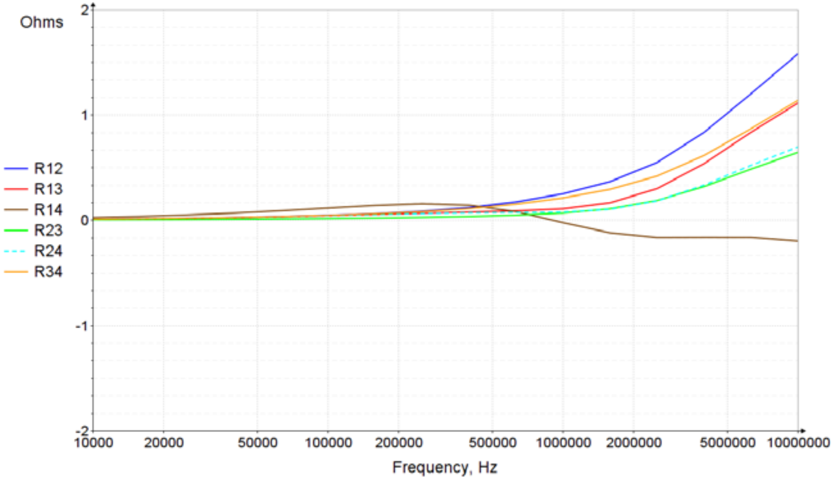
L12, L13, L24 and L34 are nearly equal.

Mutual Resistances

3 layers 2 mil Nomex

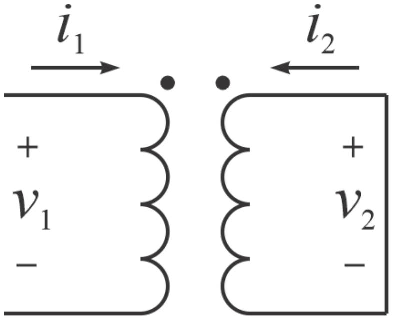


10 layers 2 mil Nomex



Definition of Leakage Impedance

Z_{leak_mn}



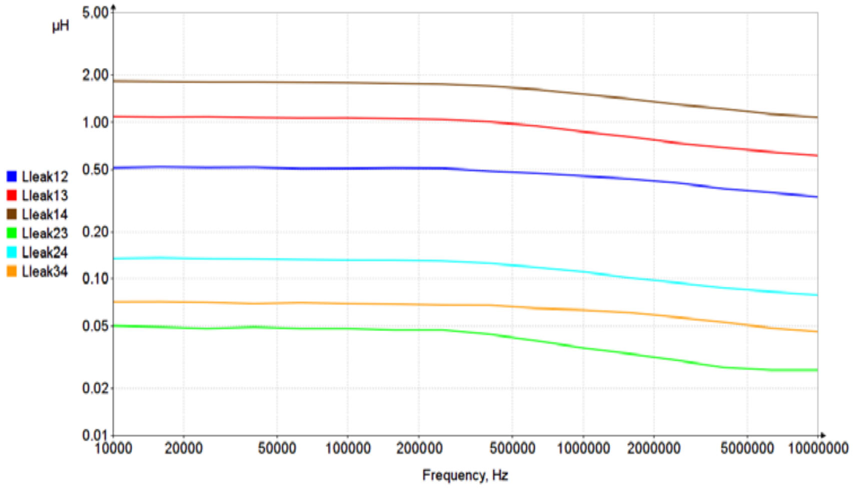
Winding m   Winding n shorted

$$Z_{leak\_mn} = Z_{mm} - \frac{{Z_{mn}}^2}{Z_{nn}}$$

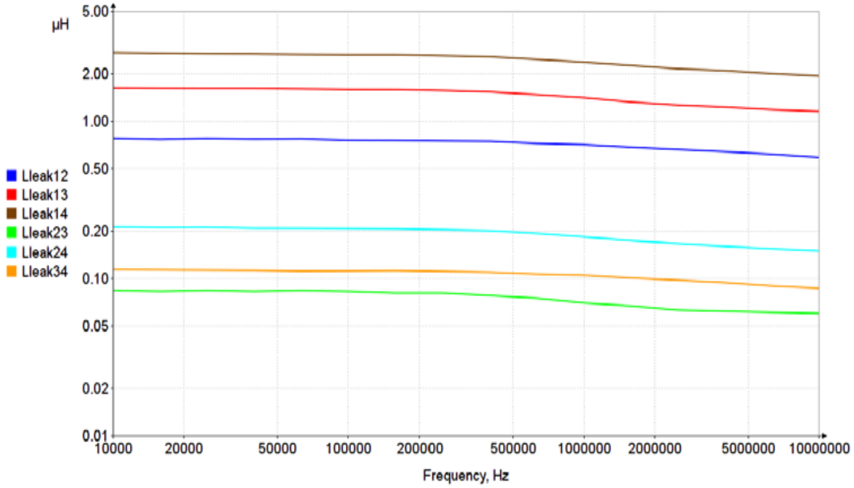
- Leakage impedance is the impedance measured at one winding when another winding is shorted
- Leakage impedances are a function of self and mutual impedances as shown in the equation above [8]
- Consequently, leakage impedances are a property of a pair of windings and generally can't be split up and assigned to individual windings when there are more than two windings

Leakage Inductances

3 layers 2 mil Nomex

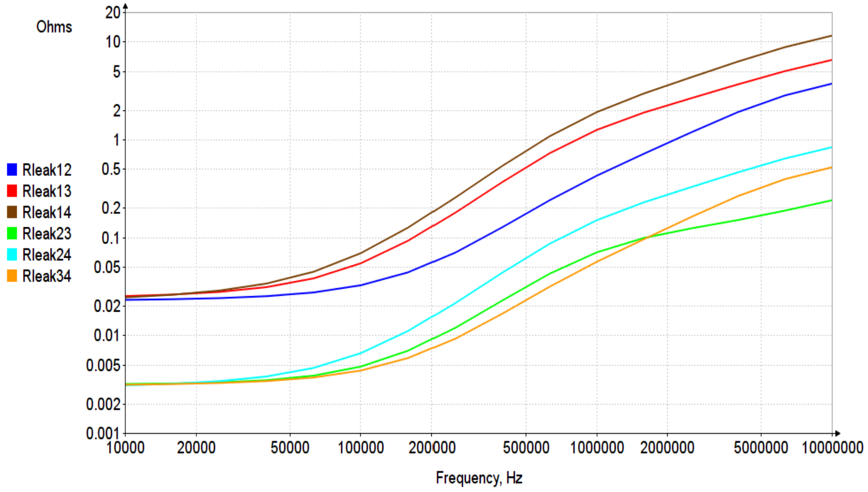


10 layers 2 mil Nomex

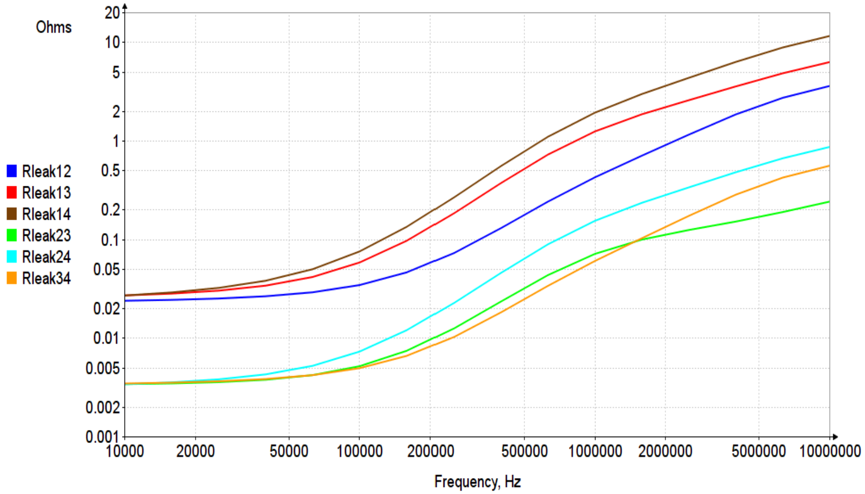


Leakage Resistances

3 layers 2 mil Nomex

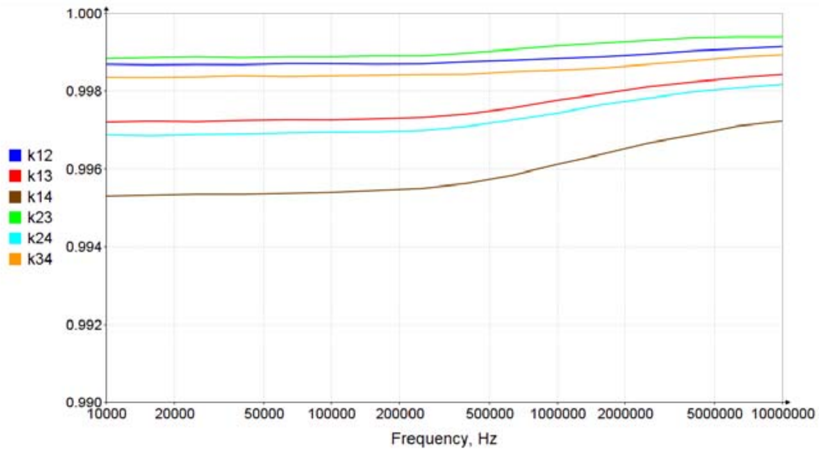


10 layers 2 mil Nomex

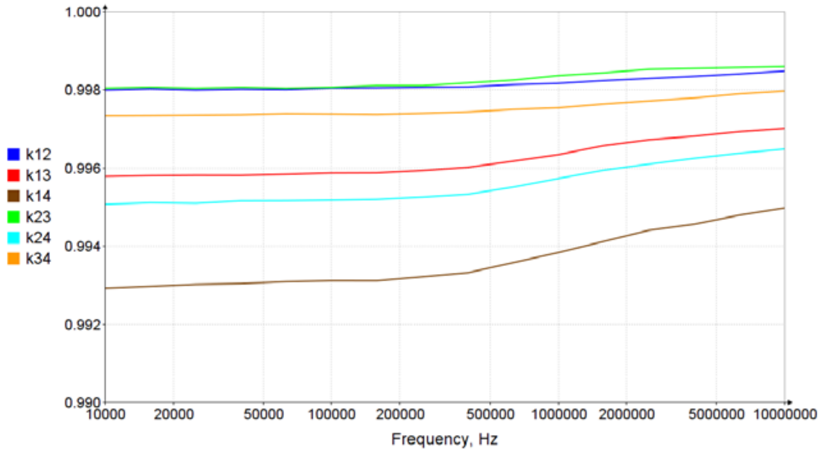


Inductive Coupling Coefficients

3 layers 2 mil Nomex



10 layers 2 mil Nomex

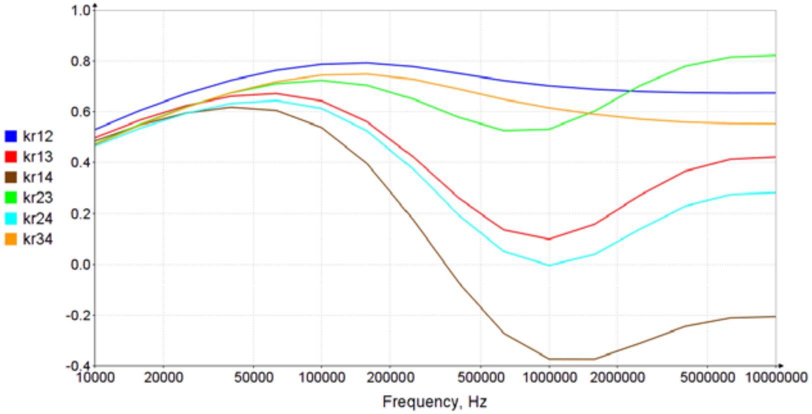


$$k_{mn} = \frac{L_{mn}}{\sqrt{L_{mm}L_{nn}}} \qquad -1 < k_{mn} < 1$$

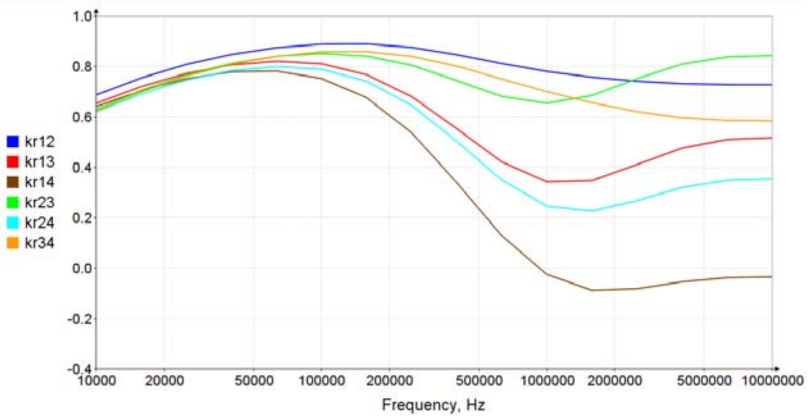
- The coupling coefficient is negative when the mutual inductance is negative
- The inequality ensures that the total stored energy is always positive for two windings
  - A more restrictive criterion is needed to ensure that the model is passive for more than two windings [9]

Resistive Coupling Coefficients

3 layers 2 mil Nomex



10 layers 2 mil Nomex

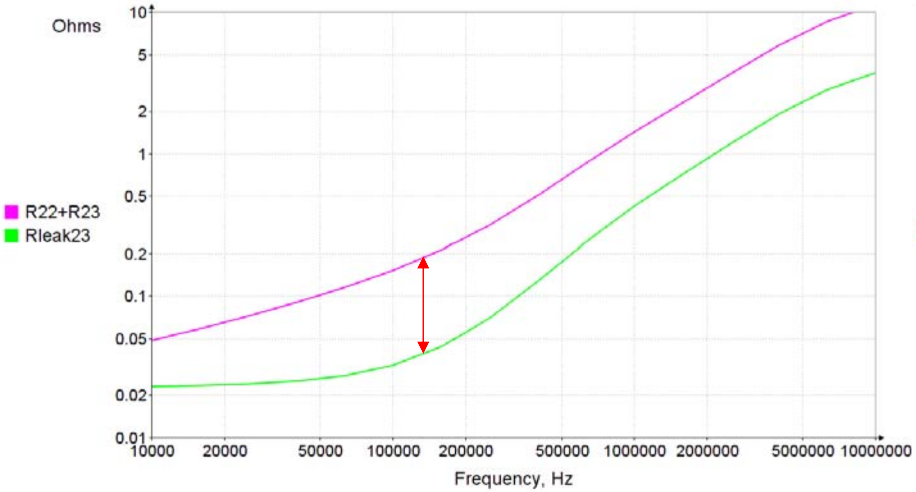


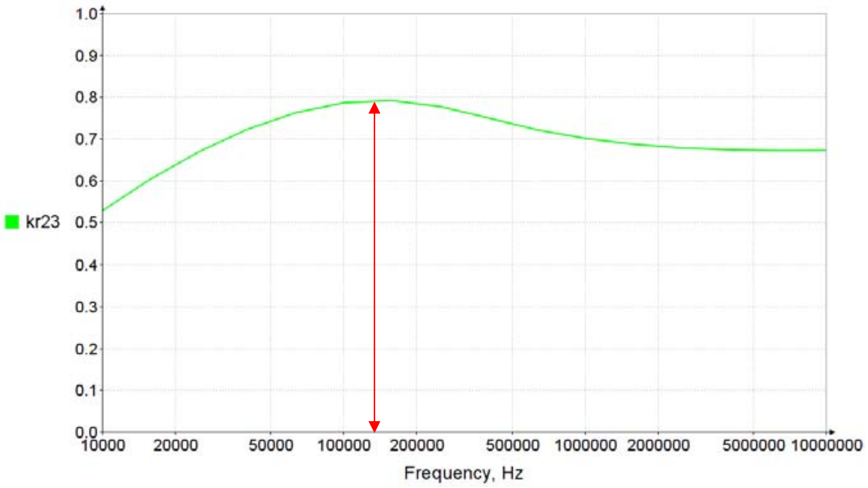
$$k^R_{mn} = \frac{R_{mn}}{\sqrt{R_{mm}R_{nn}}} \qquad -1 < k^R_{mn} < 1$$

- A coupling coefficient for mutual resistance k^R can be defined as shown above [10]
- The coupling coefficient is negative when the mutual resistance is negative
- The inequality ensures that the total dissipated power is always positive for two windings
  - I presume that the same type of constraints described in [9] also apply to resistive coupling coefficients

Effects of Mutual Resistance on Leakage Resistance

3 layers 2 mil Nomex

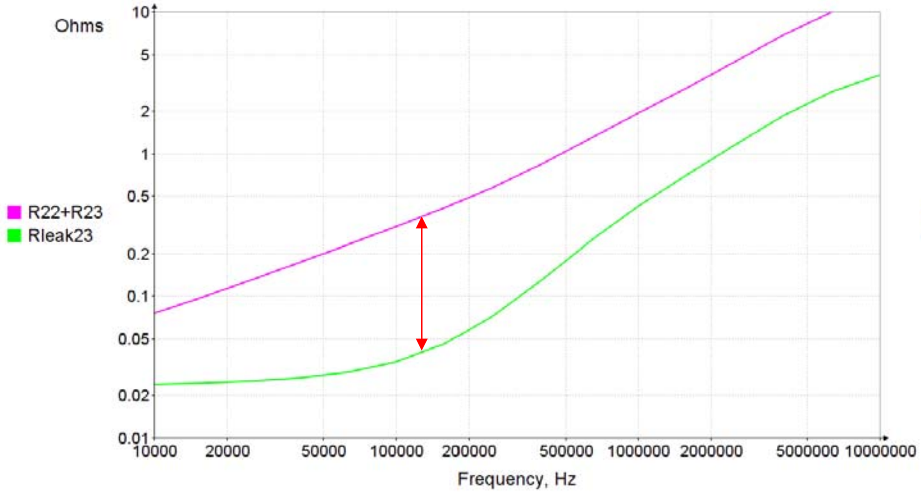


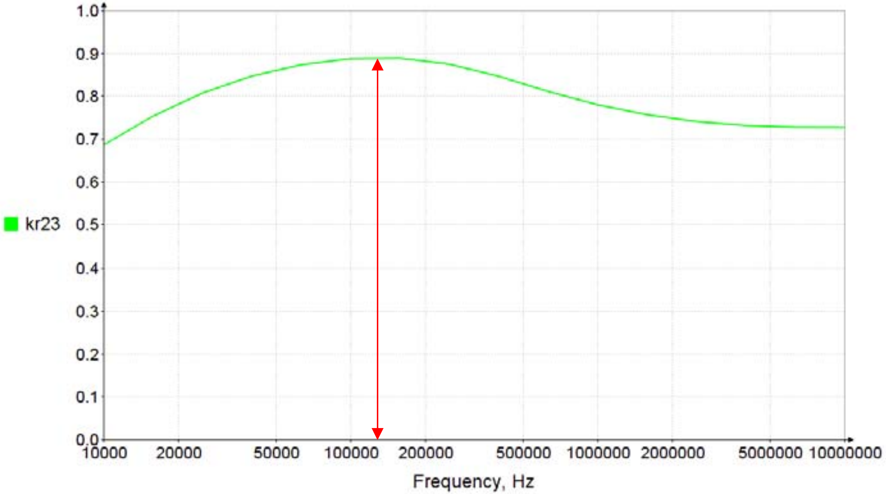


- The leakage resistance (green) is less than the sum of the self-resistance R22 and the reflected self-resistance R33
- There is a significant reduction of the ac resistance due to the mutual resistance between these adjacent windings. The reduction in leakage resistance increases as the mutual resistance coupling increases.

Effects of Mutual Resistance on Leakage Resistance

10 layers 2 mil Nomex





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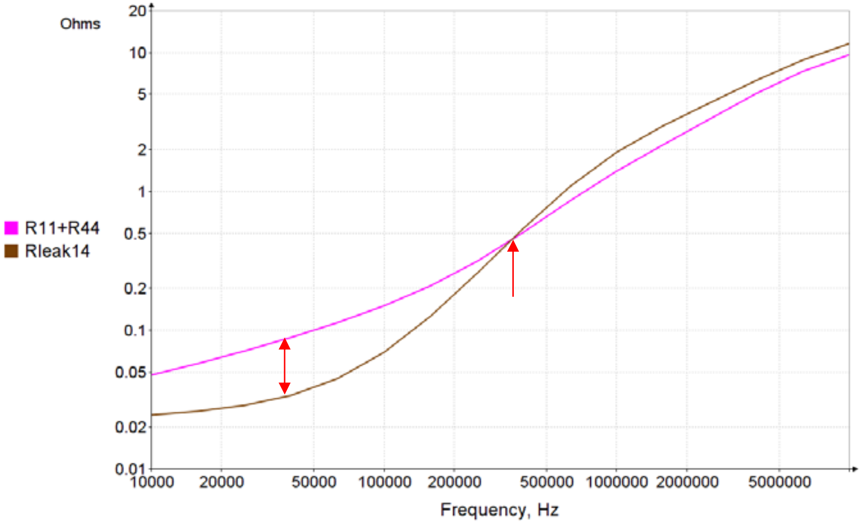
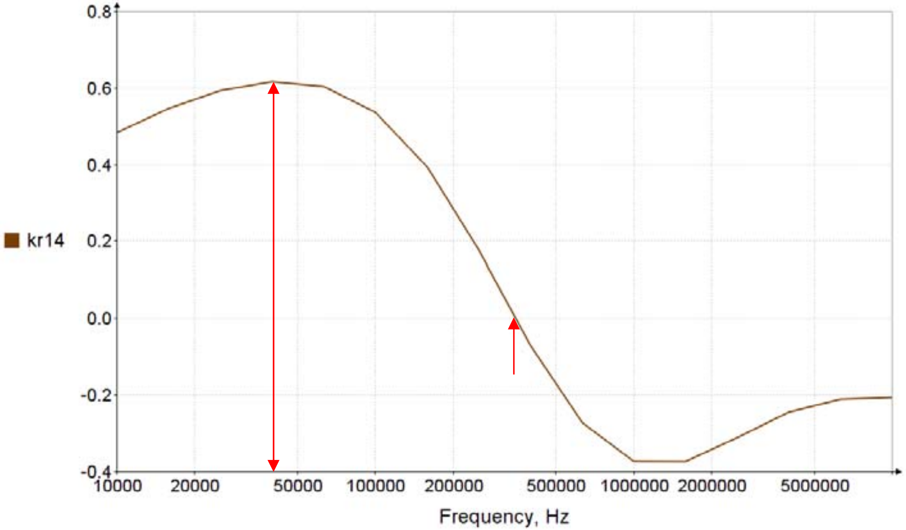
Effects of Mutual Resistance on Leakage Resistance

3 layers 2 mil Nomex

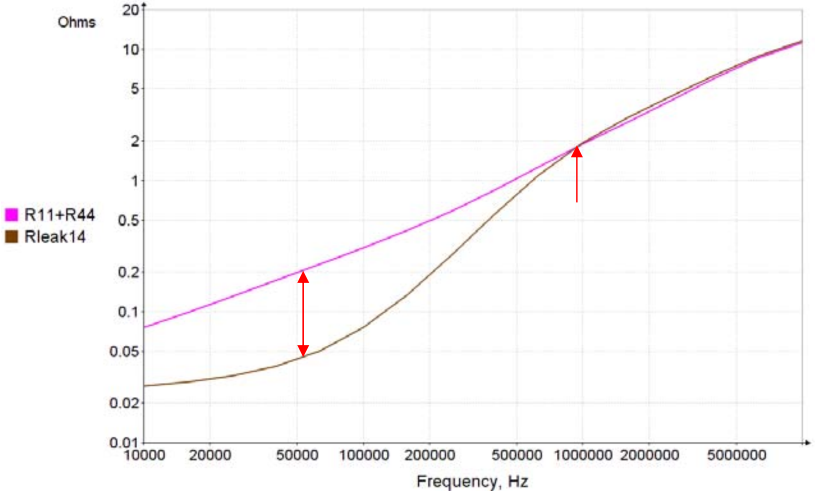


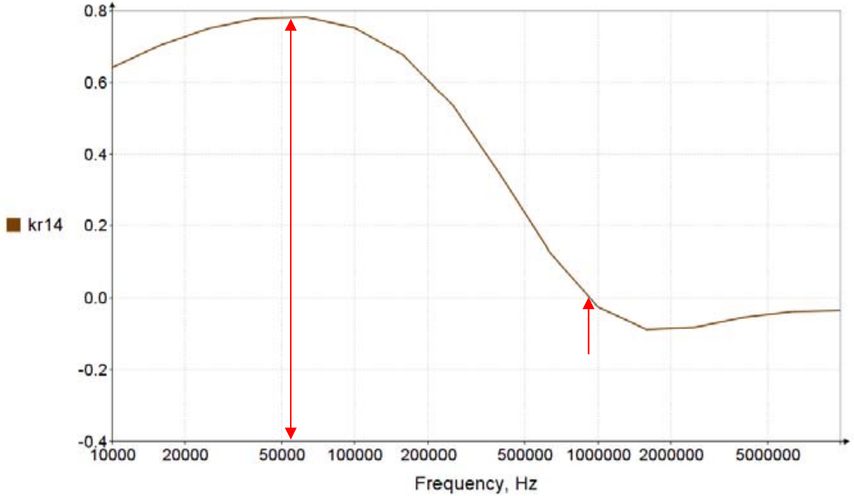


- The leakage resistance (brown) is less than the sum of the self-resistance R11 and the reflected self-resistance R44
- These windings are not adjacent. The leakage resistance increases beyond the sum of the two winding resistances when the resistive coupling changes polarity.

Effects of Mutual Resistance on Leakage Resistance

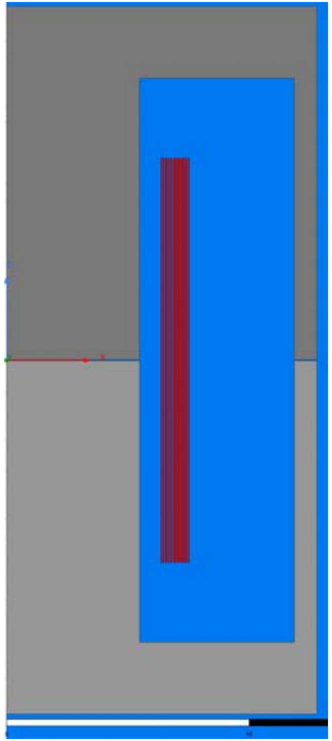
10 layers 2 mil Nomex





- The leakage resistance (brown) is less than the sum of the self-resistance R11 and the reflected self-resistance R44
- These windings are not adjacent. The leakage resistance increases beyond the sum of the two winding resistances when the resistive coupling changes polarity.

Impedance of an Isolated Winding



ANSYS Maxwell 2D Radial Model

Core: Ferroxcube ETD49-25-16 3C97

(Core loss set to zero)

Gap: 3 mil

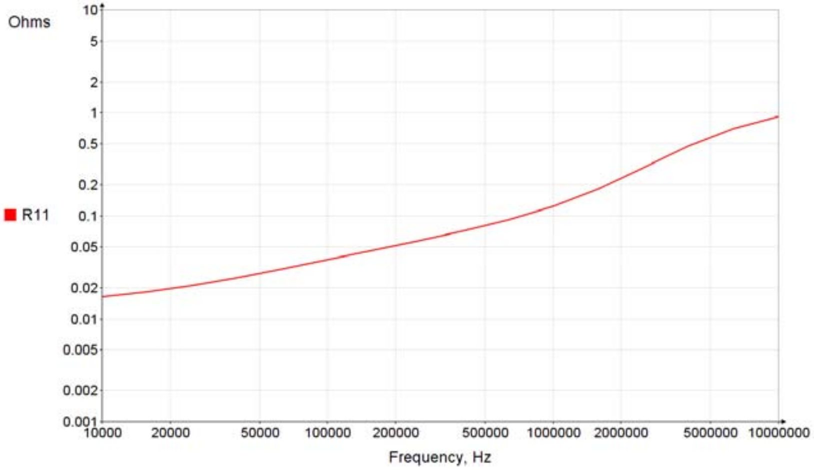
Bobbin: TDK B66368B1020T001

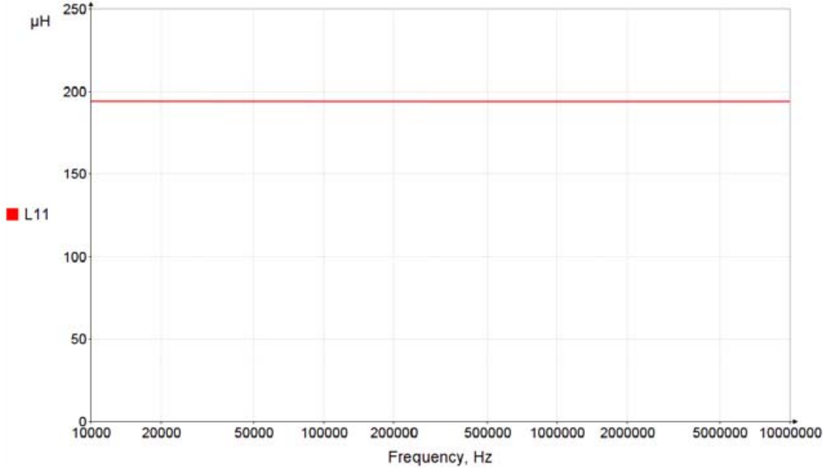
Winding: 12 Turns 0.003" x 1" copper foil

Insulation: 0.002” Nomex

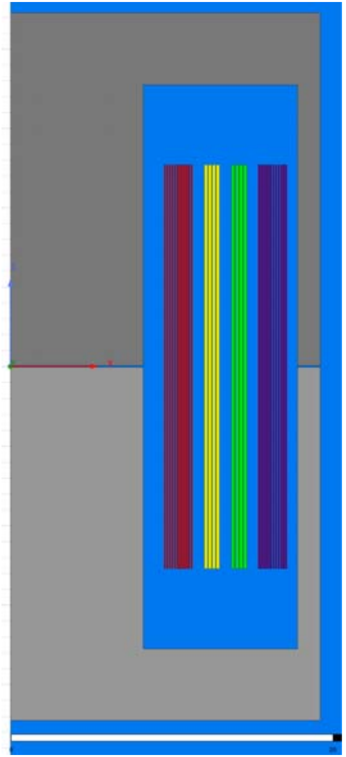
Turn-Turn spacing modeled as 0.003” based on measurements of wound bobbin

Impedance of an Isolated Winding





Impedance of Same Winding Within a Transformer



ANSYS Maxwell 2D Radial Model

Winding 1: 12 Turns 0.003" x 1" copper foil

Winding 2: 4 Turns 0.007" x 1" copper foil

Winding 3: 4 Turns 0.007" x 1" copper foil

Winding 4: 12 Turns 0.003" x 1" copper foil

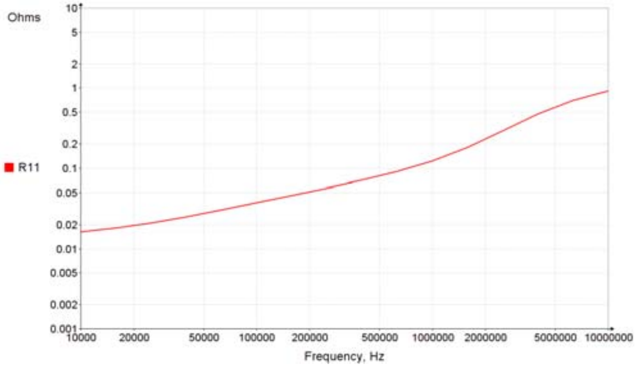
Layer Insulation: 0.002” Nomex

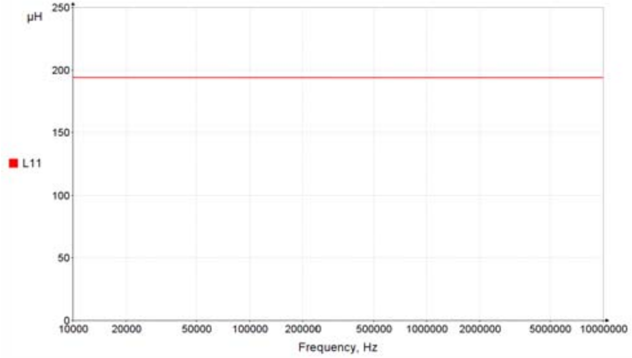
Winding Insulation: 10 Layer 0.002” Nomex

Nomex modeled as 0.003” based on measurements of wound bobbin

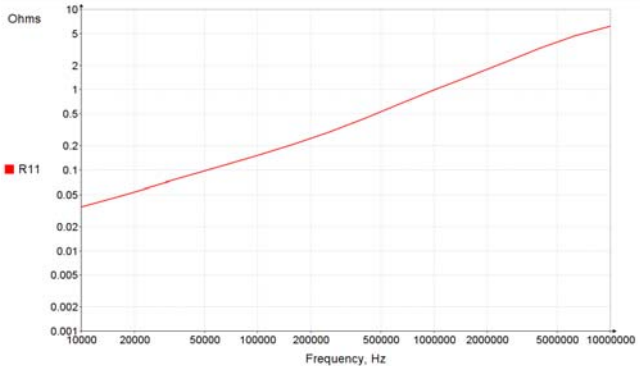
Impedance Comparison

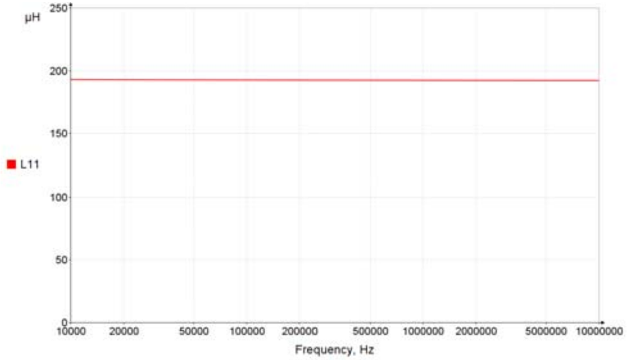
Winding by itself



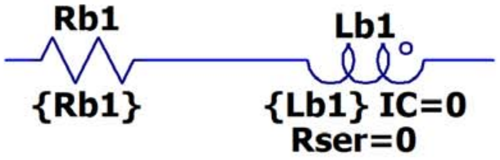


Same Winding within a transformer





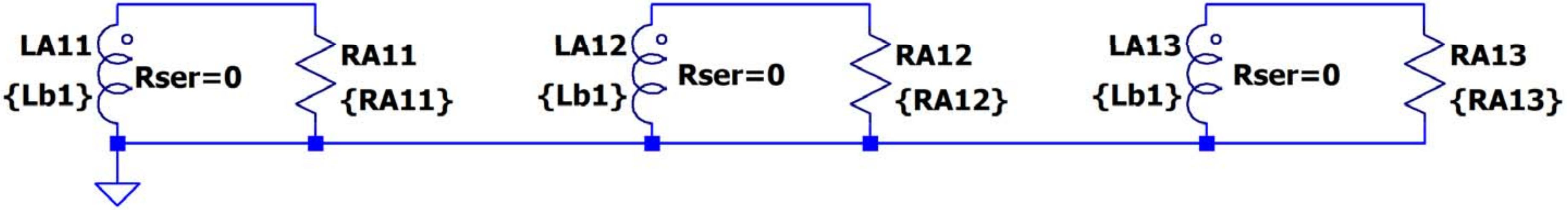
Equivalent Circuit Model of Winding Impedance




```
+       Lb1=0.00019432
+       Rb1=0.0090853
.param RA11=13.2811248049748
+       RA12=198.581714665757
+       RA13=5684.2858376726
KA1 Lb1 LA11 0.0321509449196223
KA2 Lb1 LA12 0.0155856887378975
KA3 Lb1 LA13 0.0132621479795541
```

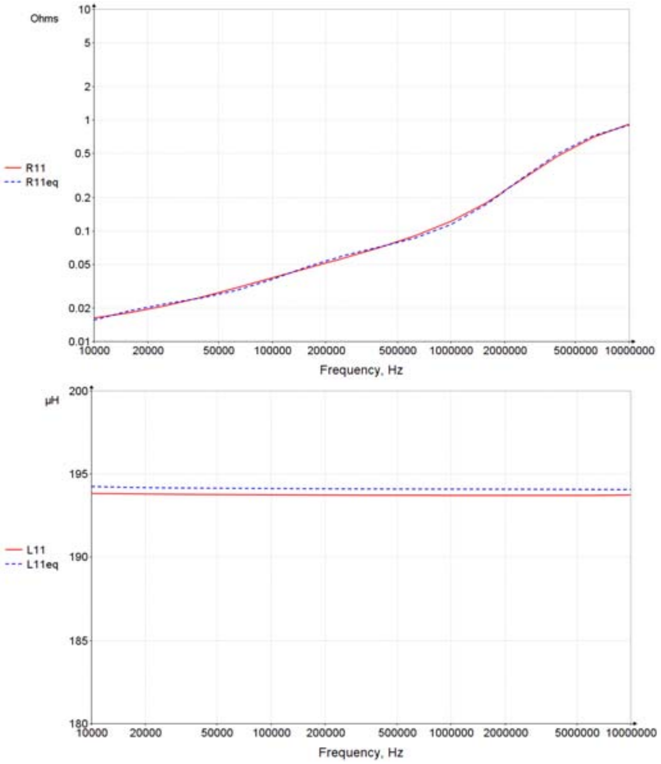
.inc ETD49-25_12T.txt

AC losses

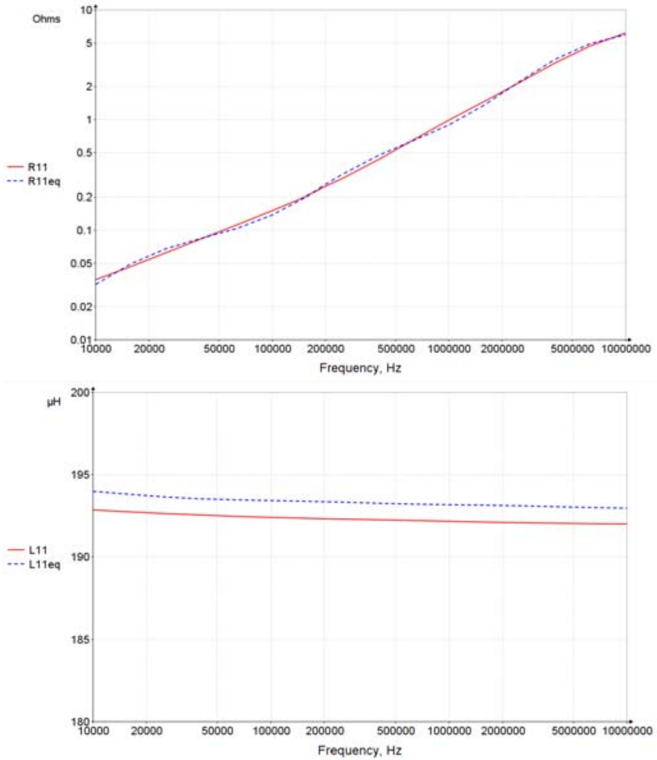


Equivalent Circuit Model of Winding Impedance

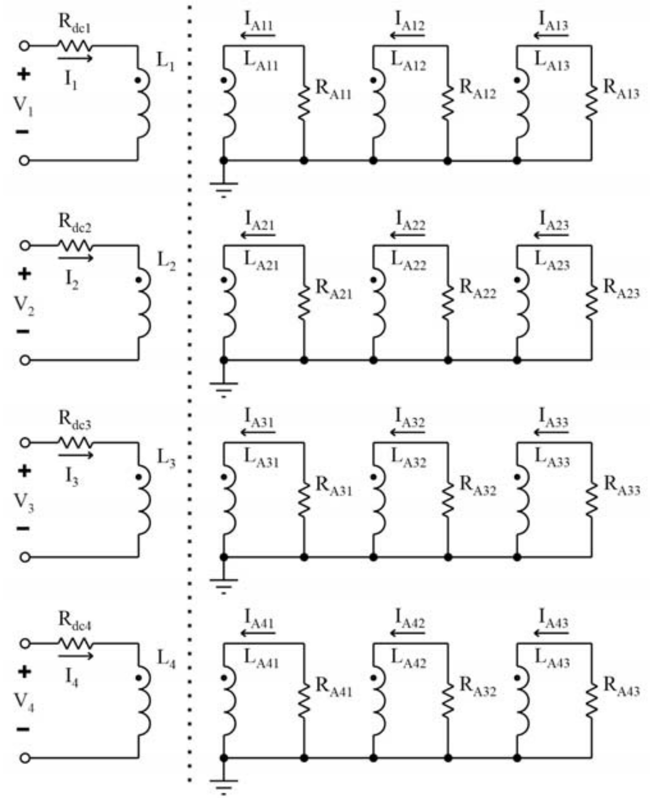
Winding by itself



Same Winding within a transformer

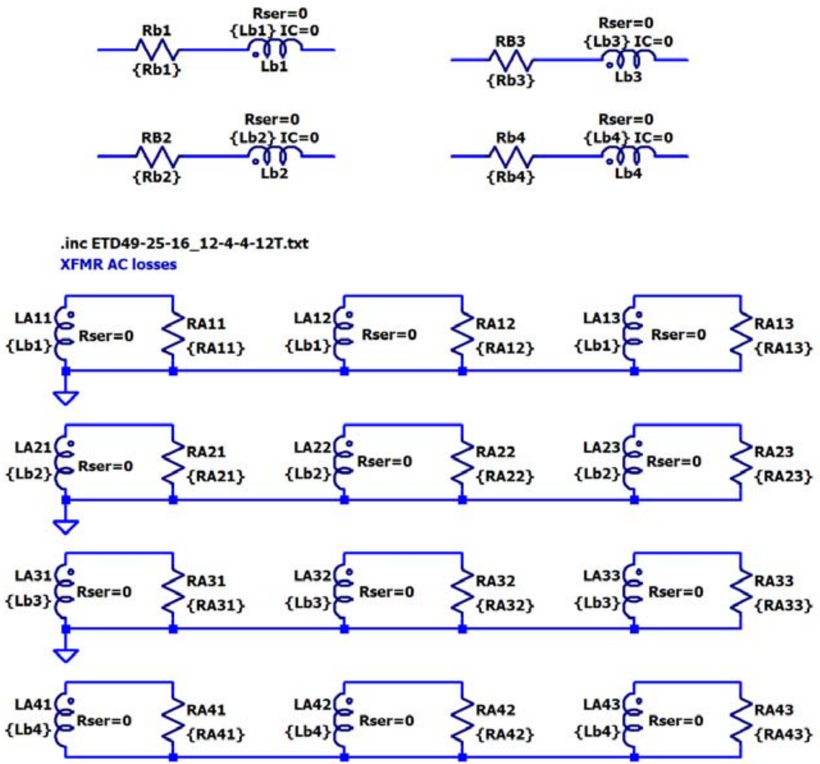


An Equivalent RL Circuit for a Four-Winding Transformer



- Model is based on methods described in [11,12]
- The physical windings are represented by L1, L2, L3 and L4
- Each physical winding is accompanied by auxiliary windings shunted by resistors that model the ac losses
- Increasing the number of auxiliary windings increases the frequency range of the model (3-aux ~ 10 MHz)
- Each physical winding is coupled to each of the auxiliary windings
- Some of the couplings could be negative
- The auxiliary windings have the same inductance as their associated physical winding
- The parameter values were determined by a solver in Mathcad that attempts to match the performance of the model to the impedance matrix data imported from Maxwell

Transformer Equivalent Circuit Model




```
.param Lb1=0.0001942
+      Lb2=2.1581e-05
+      Lb3=2.1575e-05
+      Lb4=0.00019399
.param Rb1=0.0090854
+      Rb2=0.0015571
+      Rb3=0.0017664
+      Rb4=0.01418
.param RA11=6298.48735893051
+      RA12=7.82033324539965
+      RA13=1099.86211174956
+      RA21=2.58392936460871
+      RA22=19.9782676625286
+      RA23=714.052153435295
+      RA31=4.74552746006711
+      RA32=93.2466219025053
+      RA33=1147.41553865716
+      RA41=990.880353341061
+      RA42=689.178480736719
+      RA43=6874.80515163138
KA1 Lb1 LA11 0.0174753852251423
KA2 Lb1 LA12 0.0600395501685122
KA3 Lb1 LA13 0.0259267489912189
KA4 Lb1 LA21 0.0102497019739799
KA5 Lb1 LA22 0.0261608216852448
KA6 Lb1 LA23 0.00737161407564278
KA7 Lb1 LA31 0.0374917853799379
KA8 Lb1 LA32 -0.00511830441554691
KA9 Lb1 LA33 -0.0117244222029584
KA10 Lb1 LA41 0.00674338243958032
KA11 Lb1 LA42 0.0153369384144078
KA12 Lb1 LA43 -0.0211255520600343
KA13 Lb2 LA11 -0.00407636485303306
KA14 Lb2 LA12 0.0612265652683242
KA15 Lb2 LA13 0.017060838144862
KA16 Lb2 LA21 0.00428842568705198
KA17 Lb2 LA22 0.027129657434239
KA18 Lb2 LA23 0.0170200265739508
KA19 Lb2 LA31 0.0377309178709713
KA20 Lb2 LA32 0.00659413200674197
KA21 Lb2 LA33 -0.00719794084932156
KA22 Lb2 LA41 0.0200791055468109
KA23 Lb2 LA42 0.00805373794823475
KA24 Lb2 LA43 -0.0278146234646167
KA25 Lb3 LA11 -0.000909264271214426
KA26 Lb3 LA12 0.061047766316242
KA27 Lb3 LA13 -0.0121169490727813
KA28 Lb3 LA21 -0.00439854443574395
KA29 Lb3 LA22 0.0272361149345052
KA30 Lb3 LA23 0.0182109194950555
KA31 Lb3 LA31 0.0377405288713751
KA32 Lb3 LA32 -0.00385677458622876
KA33 Lb3 LA33 0.00597414079828727
KA34 Lb3 LA41 0.0226614916107693
KA35 Lb3 LA42 0.00779219337346387
KA36 Lb3 LA43 -0.0275666211705724
KA37 Lb4 LA11 -0.0124709571098259
KA38 Lb4 LA12 0.0592117185958606
KA39 Lb4 LA13 -0.0238588649629732
KA40 Lb4 LA21 -0.0126287025191818
KA41 Lb4 LA22 0.0260993156183912
KA42 Lb4 LA23 -0.00761399596581985
KA43 Lb4 LA31 0.0374342121749694
KA44 Lb4 LA32 0.00597872288177876
KA45 Lb4 LA33 0.0110593303205595
KA46 Lb4 LA41 0.00715813706493723
KA47 Lb4 LA42 0.0159117145488322
KA48 Lb4 LA43 -0.0201749309394903
Kb1 Lb1 Lb2 0.997973892475517
Kb2 Lb1 Lb3 0.99567171106925
Kb3 Lb1 Lb4 0.99270990079083
Kb4 Lb2 Lb3 0.998007239231169
Kb5 Lb2 Lb4 0.994943764205363
Kb6 Lb3 Lb4 0.997292501813081
```

Model Coupling Coefficients

Physical Windings

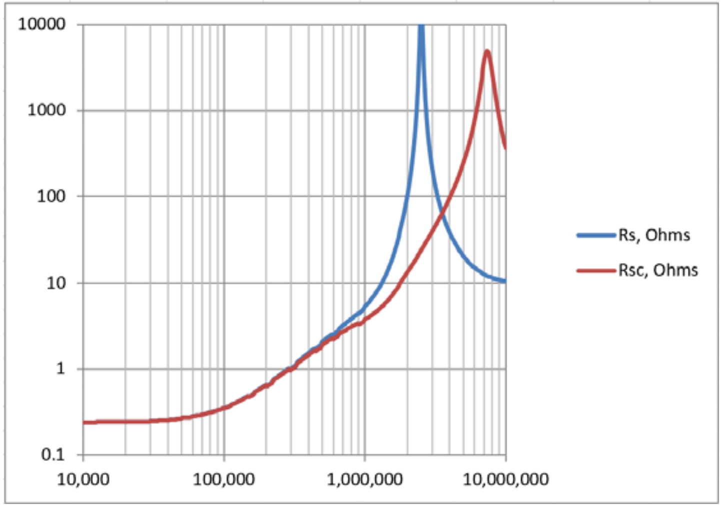
$$K_{mod}(k_A) = \begin{bmatrix}
\boxed{\begin{matrix} 1 & 0.993 & 0.979 & 0.966 \\ 0.993 & 1 & 0.99 & 0.976 \\ 0.979 & 0.99 & 1 & 0.99 \\ 0.966 & 0.976 & 0.99 & 1 \end{matrix}} & \begin{matrix} -0.014 & 0.018 & 0.055 & -1.783\cdot10^{-5} & -0.033 & 0.05 & -3.819\cdot10^{-5} & -0.009 & 0.114 & 0.07 & -0.014 & 0.019 \\ 0.023 & 0.006 & 0.059 & 0.003 & -0.089 & 0.054 & 0.003 & 0.01 & 0.099 & 0.047 & 0.014 & 0.007 \\ -0.008 & -0.023 & 0.056 & 7.623\cdot10^{-6} & -0.105 & 0.051 & 6.891\cdot10^{-5} & -0.013 & 0.101 & -0.036 & 0.016 & -0.022 \\ 0.036 & -0.041 & 0.05 & 0.004 & -0.054 & 0.045 & -0.004 & 0.012 & 0.114 & -0.085 & -0.013 & -0.039 \end{matrix} \\
\begin{matrix} -0.014 & 0.023 & -0.008 & 0.036 \\ 0.018 & 0.006 & -0.023 & -0.041 \\ 0.055 & 0.059 & 0.056 & 0.05 \\ -1.783\cdot10^{-5} & 0.003 & 7.623\cdot10^{-6} & 0.004 \\ -0.033 & -0.089 & -0.105 & -0.054 \\ 0.05 & 0.054 & 0.051 & 0.045 \\ -3.819\cdot10^{-5} & 0.003 & 6.891\cdot10^{-5} & -0.004 \\ -0.009 & 0.01 & -0.013 & 0.012 \\ 0.114 & 0.099 & 0.101 & 0.114 \\ 0.07 & 0.047 & -0.036 & -0.085 \\ -0.014 & 0.014 & 0.016 & -0.013 \\ 0.019 & 0.007 & -0.022 & -0.039 \end{matrix} & \begin{matrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{matrix}
\end{bmatrix}$$

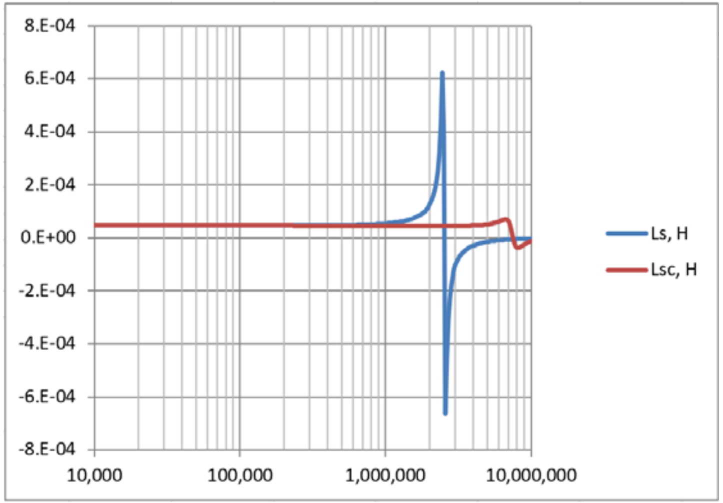
$$\text{eigenvals}\left(K_{mod}(k_A)\right) = \begin{bmatrix} 3.976231 \\ 1.021110 \\ 1.003832 \\ 1.001675 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 1.000000 \\ 0.970672 \\ 0.019848 \\ 0.004485 \\ 0.002148 \end{bmatrix}$$

- The couplings among the physical windings are high, but the auxiliary couplings are low
- A matrix of all the coupling coefficients used in the model can be used to check the stability of the model [9]
- All the eigenvalues of the coupling matrix must non-negative to ensure stability of the model
- The solver checks for stability and rejects unstable solutions
- The model is reduced-order because not all the couplings are included (the model in [11] includes all couplings)

Self Resistance and Inductance Capacitive Correction

Original Series Inductance	Original Series Resistance	Radian Frequency	Series Inductor Reactace	Equivalent Parallel Inductance Reactance	Parallel Resistance	*Cp* Reactance	Corrected Parallel Inductor Reactance	Corrected Series Inductor Reactance	Corrected Series Inductor Resistance	Corrected Series Inductance
L_s	R_s	$2\pi f$	ωL_s	$\frac{X_{Ls}{}^2+R_s{}^2}{X_{Ls}}$	$\frac{X_{Ls}{}^2+R_s{}^2}{R_s}$	$\frac{-1}{\omega C_p}$	$\frac{1}{\frac{1}{X_{Lp}}-\frac{1}{X_{Cp}}}$	$\frac{R_p{}^2 X_{Lpc}}{X_{Lpc}{}^2+R_p{}^2}$	$\frac{R_p X_{Lpc}{}^2}{X_{Lpc}{}^2+R_p{}^2}$	$\frac{X_{Lsc}}{\omega}$
Results		ω	X_{Ls}	X_{Lp}	R_p	X_{Cp}	X_{Lpc}	X_{Lsc}	R_{sc}	L_{sc}





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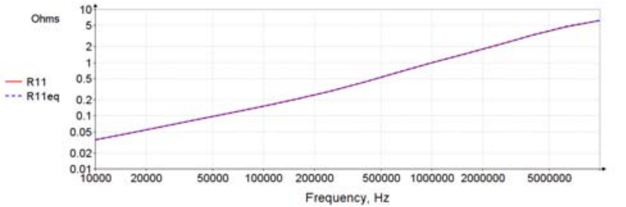
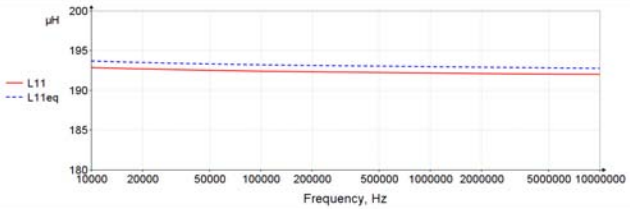
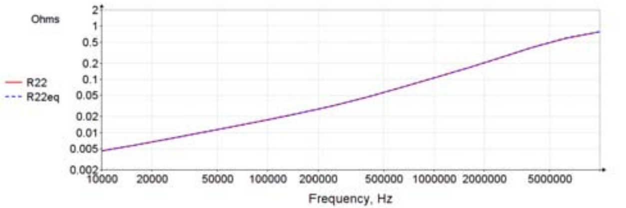
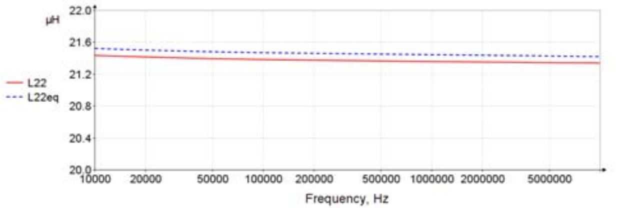
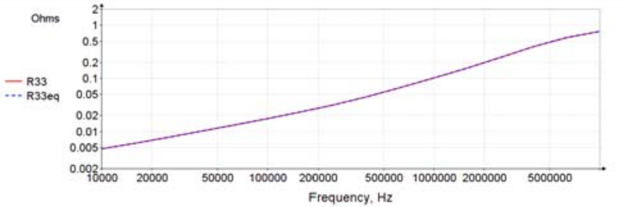
FEA and Equivalent Circuit Self Resistances and Inductances

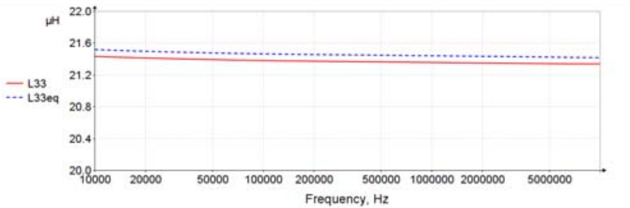


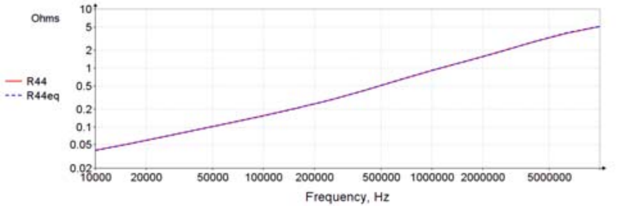


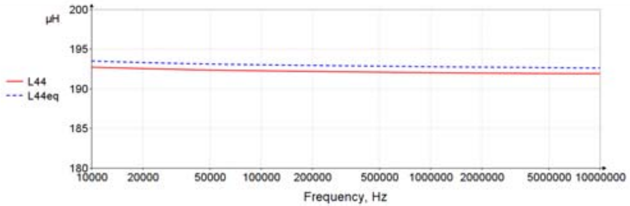




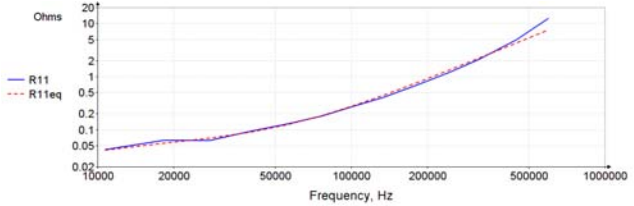


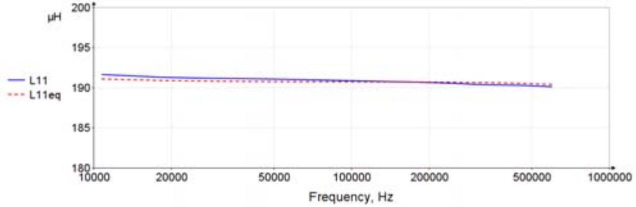


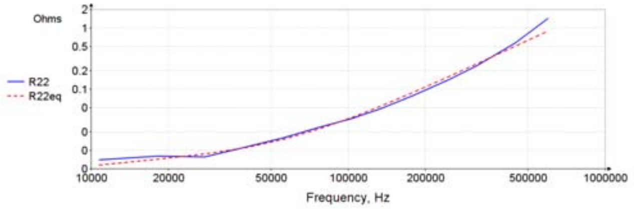


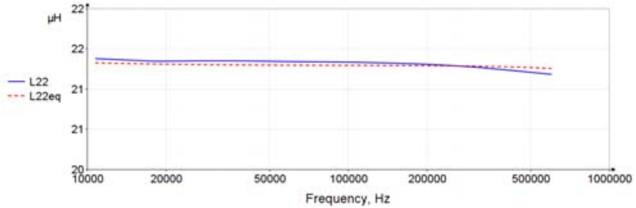


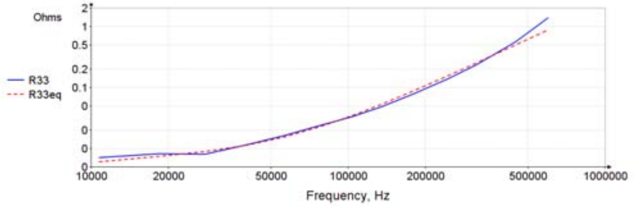
Measured and Eq. Cir. Self Resistances and Inductances

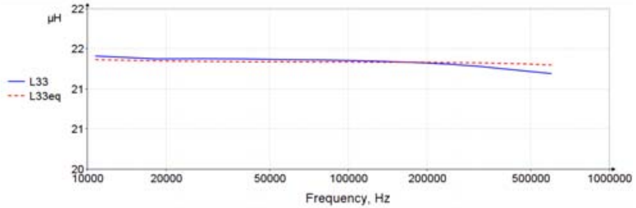


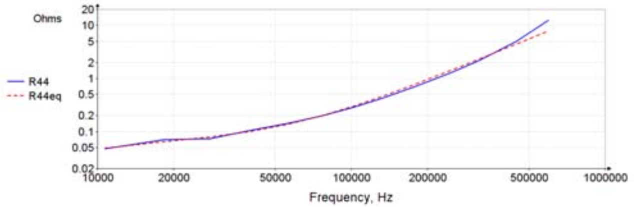


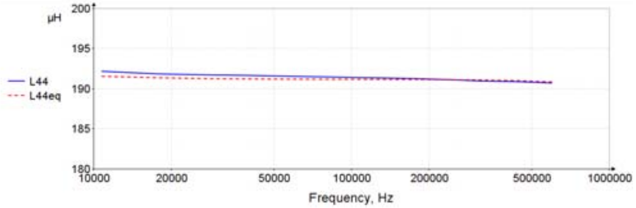




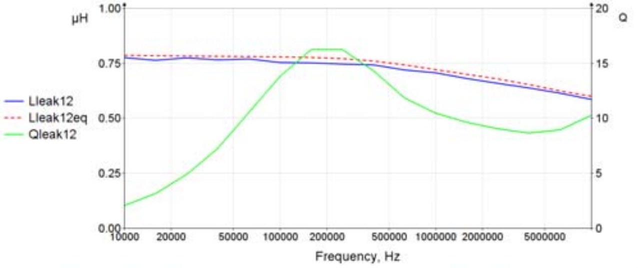


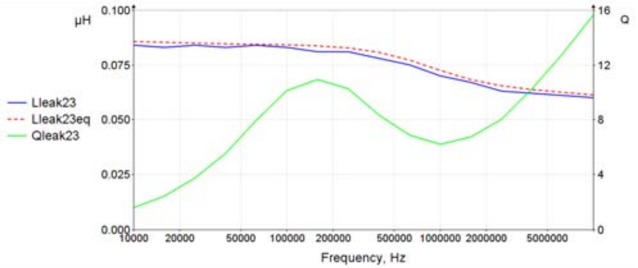




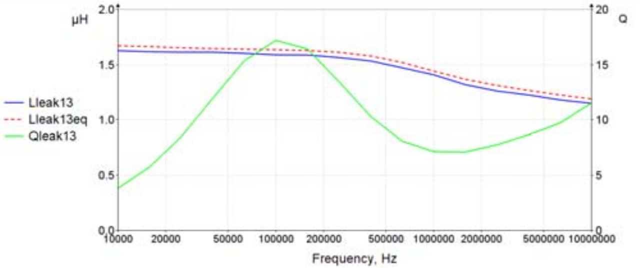


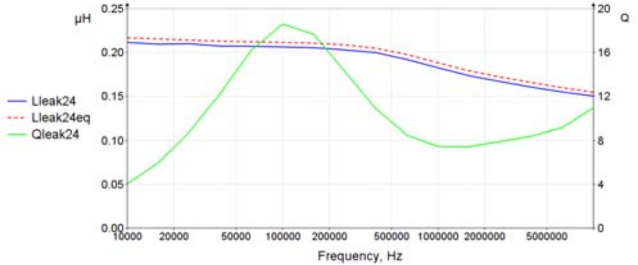
FEA and Equivalent Circuit Leakage Inductances

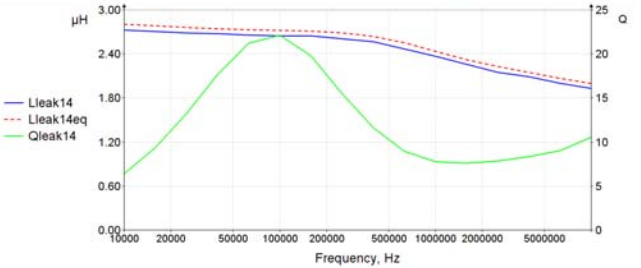


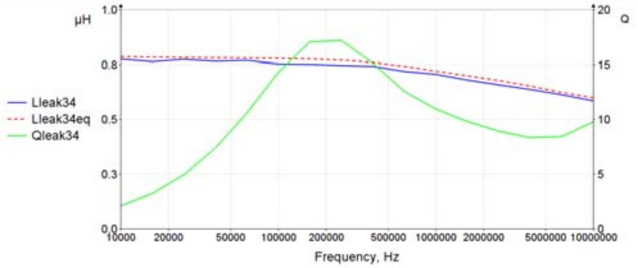


$$Q = \frac{Im(Zleak)}{Re(Zleak)}$$

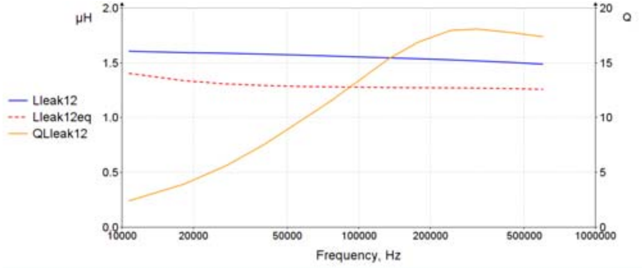


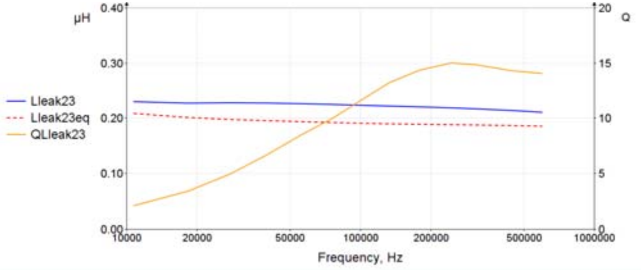




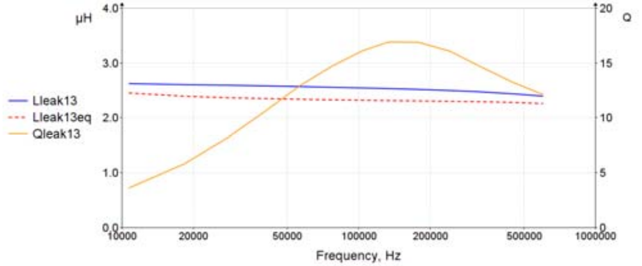


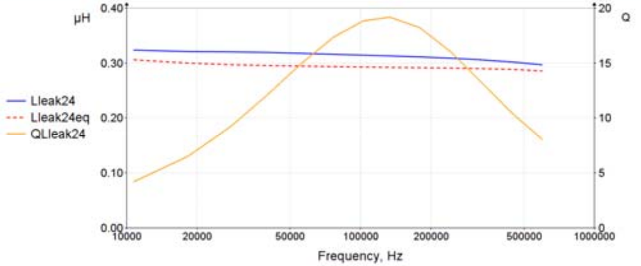
Measured and Equivalent Circuit Leakage Inductances

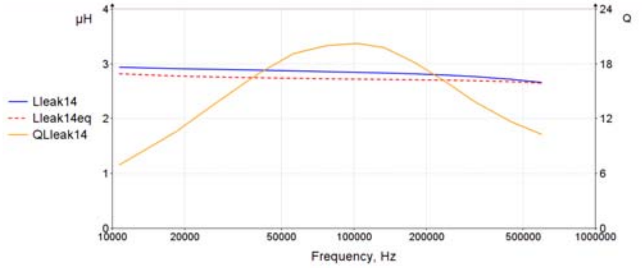


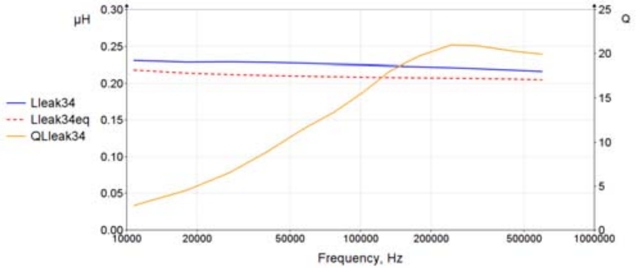


$$Q = \frac{Im(Zleak)}{Re(Zleak)}$$

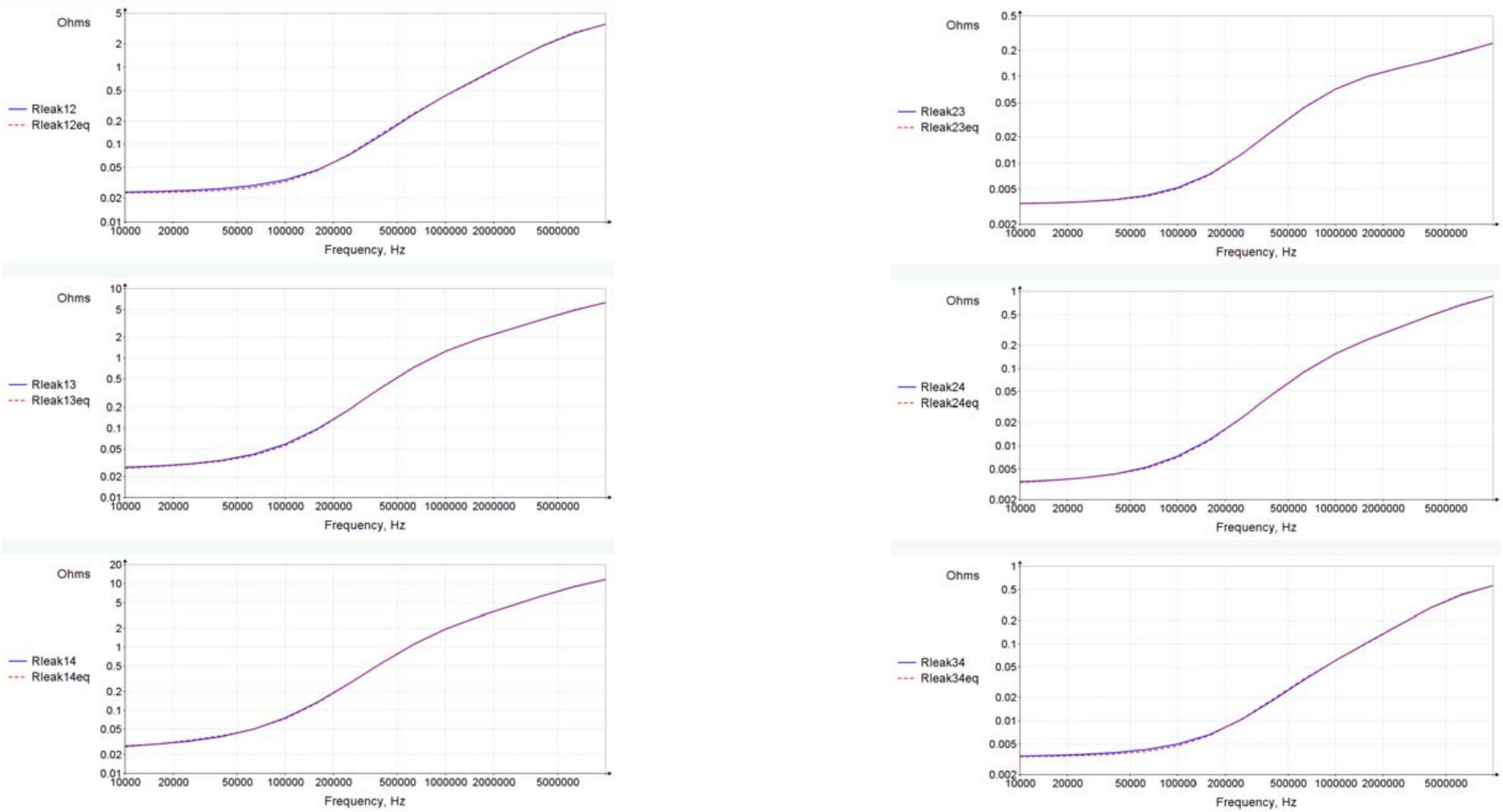




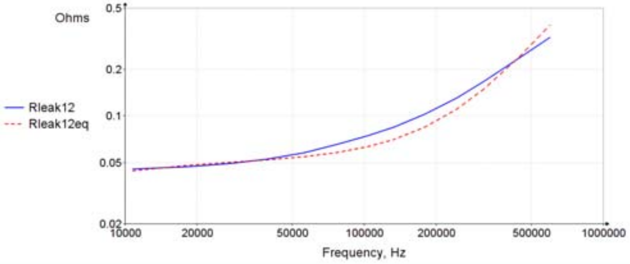


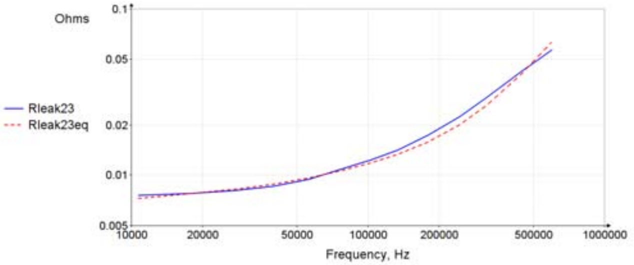


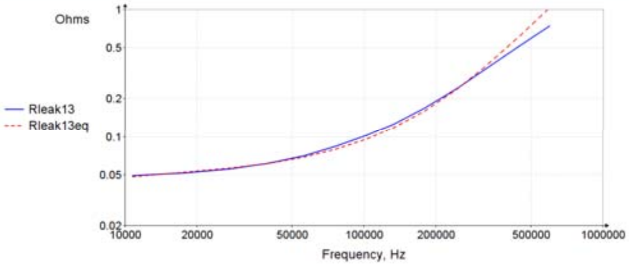
FEA and Equivalent Circuit Leakage Resistances

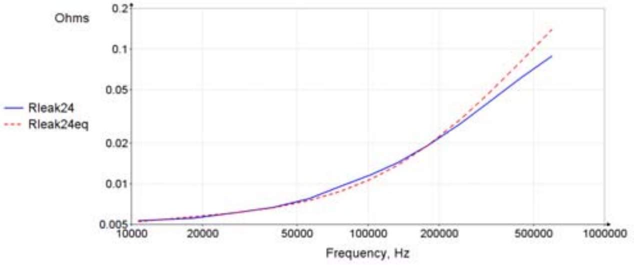


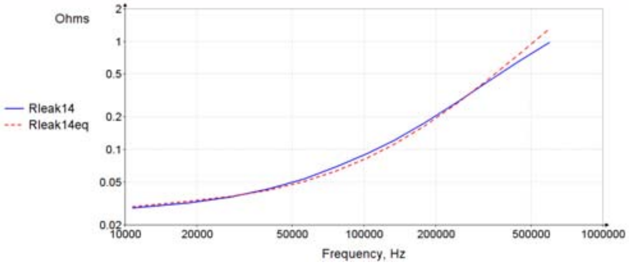
Measured and Equivalent Circuit Leakage Resistances

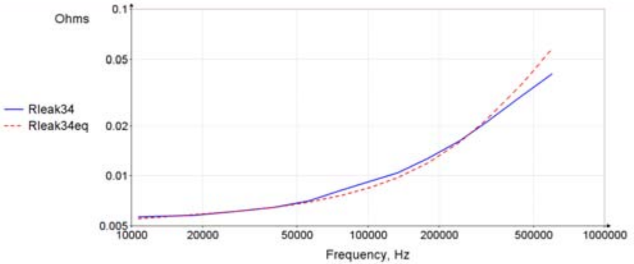




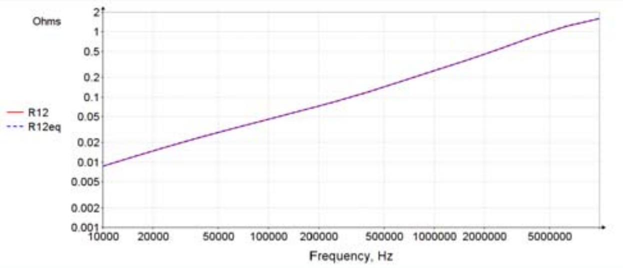


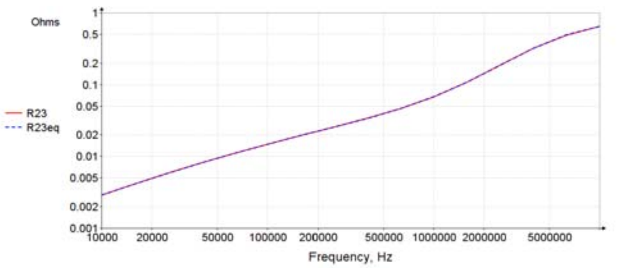


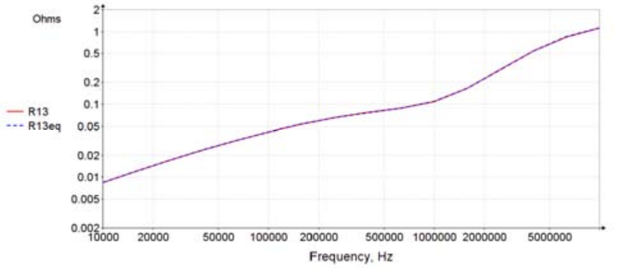


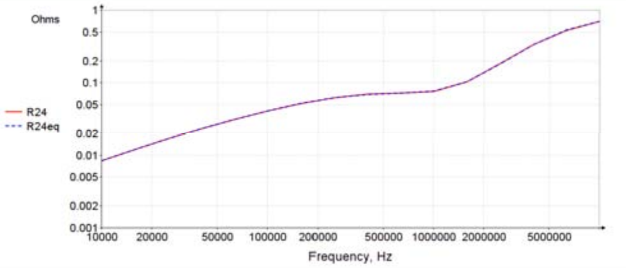


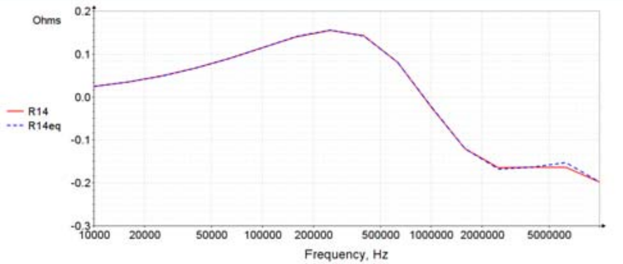
FEA and Equivalent Circuit Mutual Resistances

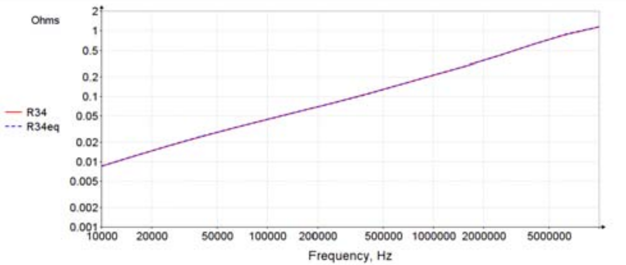




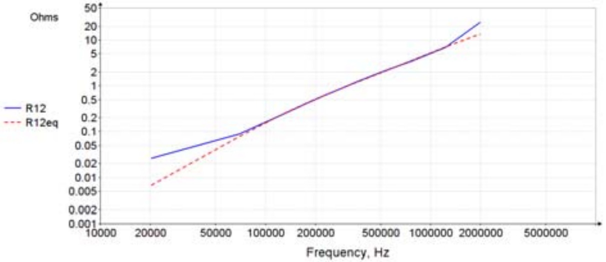


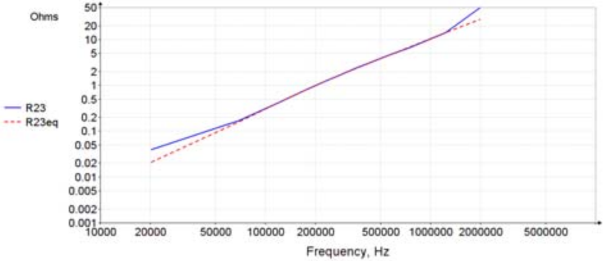


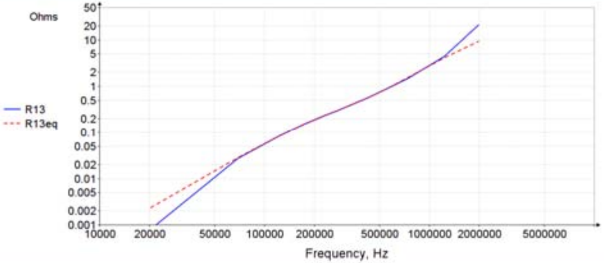


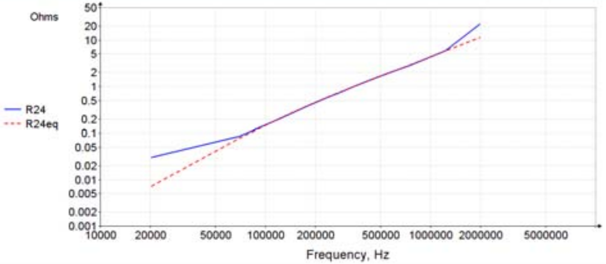


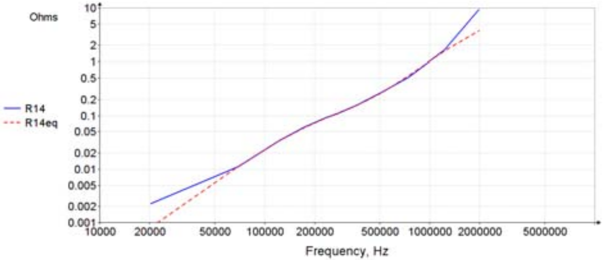
Measured and Equivalent Circuit Mutual Resistances

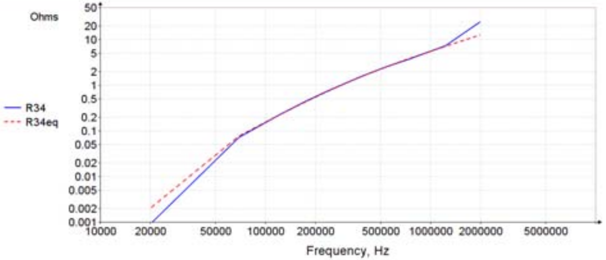




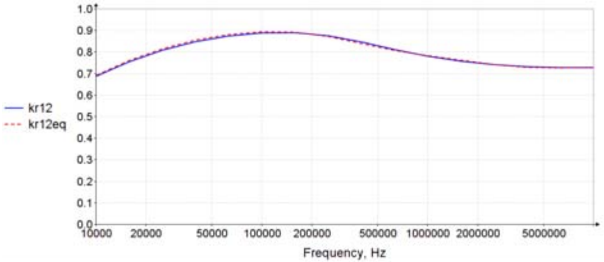


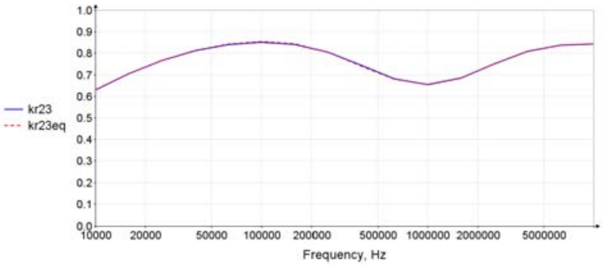


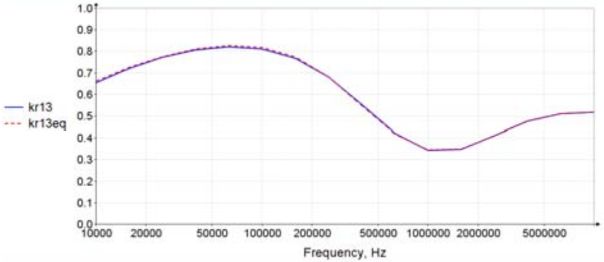


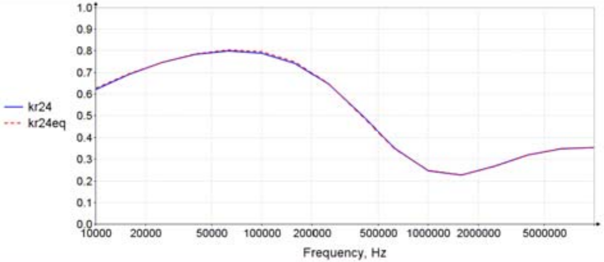


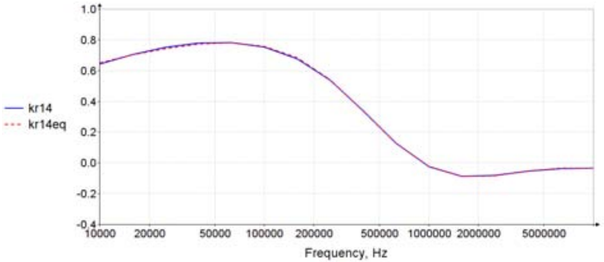
FEA and Equivalent Circuit Mutual Resistance Coupling

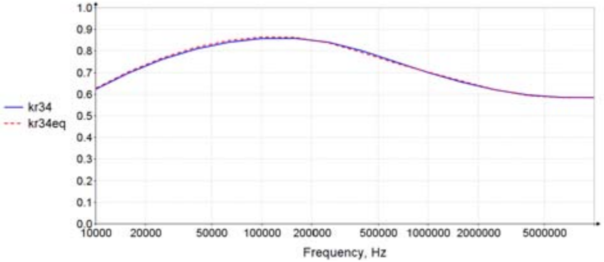




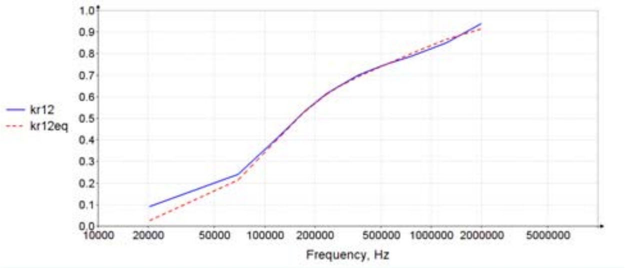


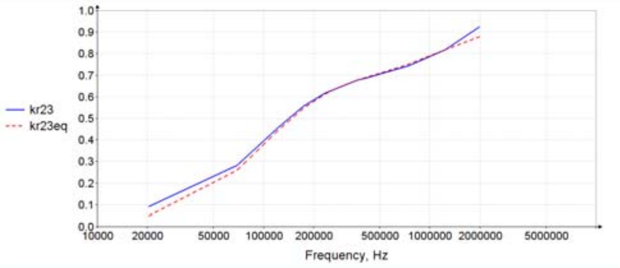


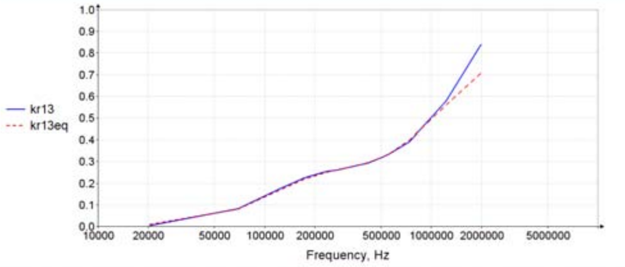


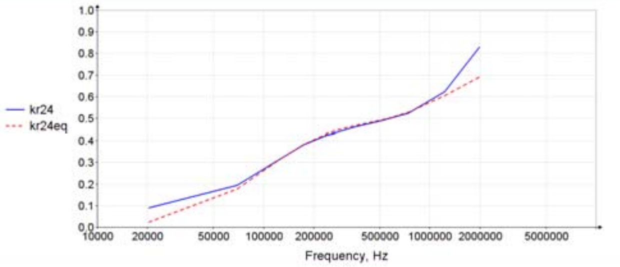


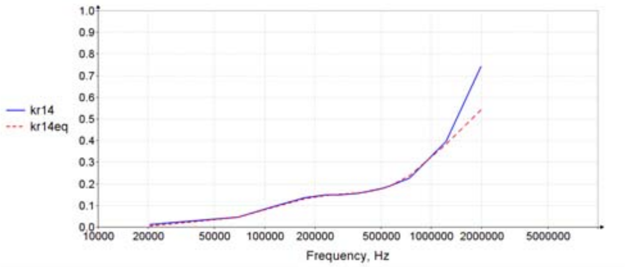
Measured and Equivalent Circuit Mutual Resistance Coupling

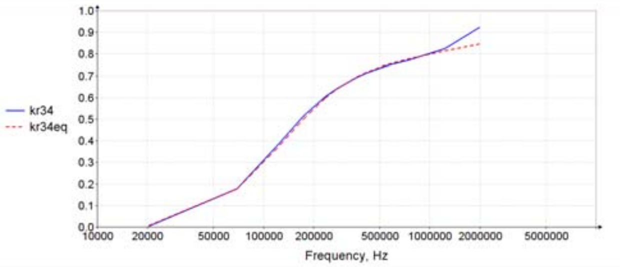




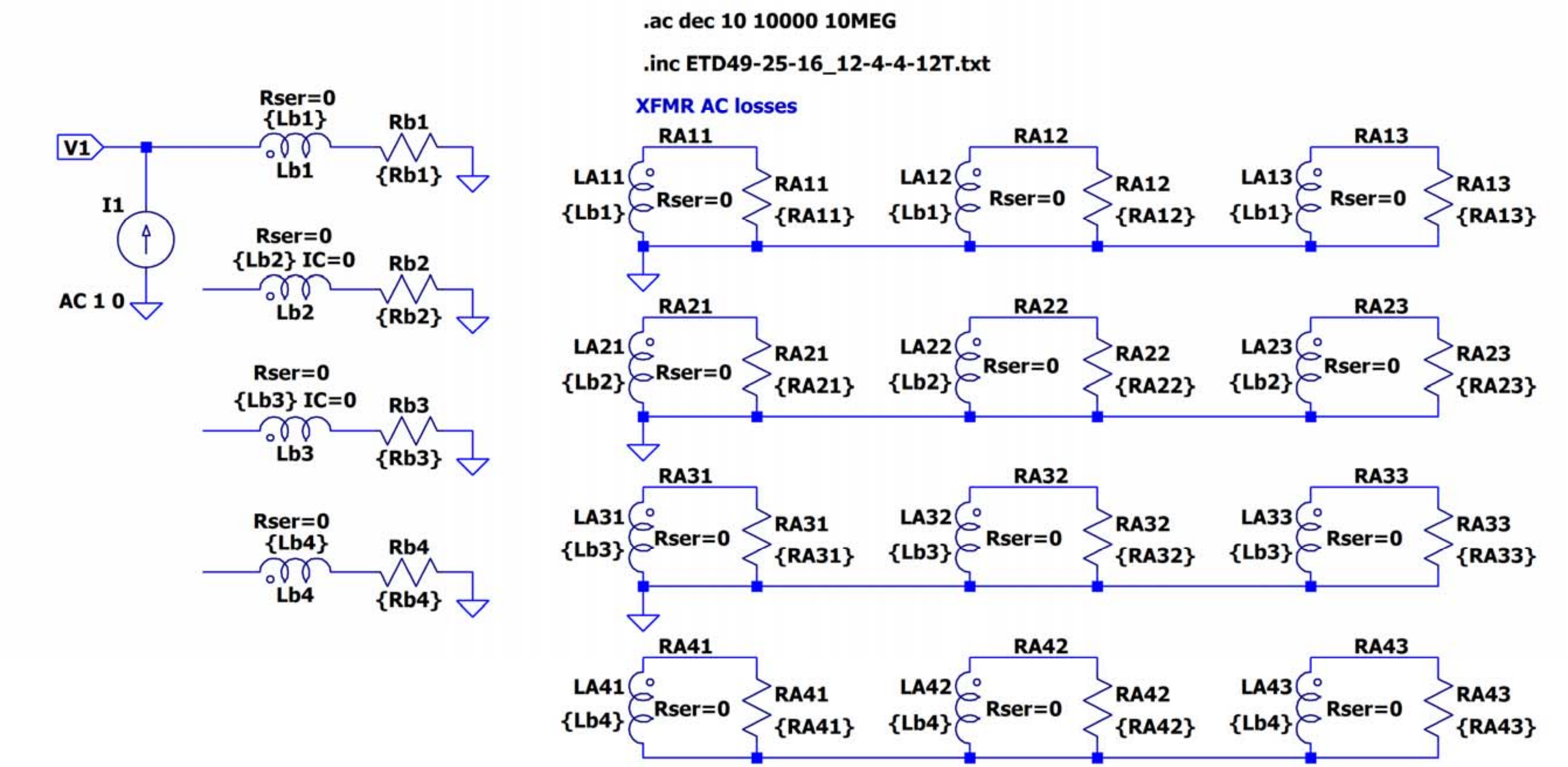






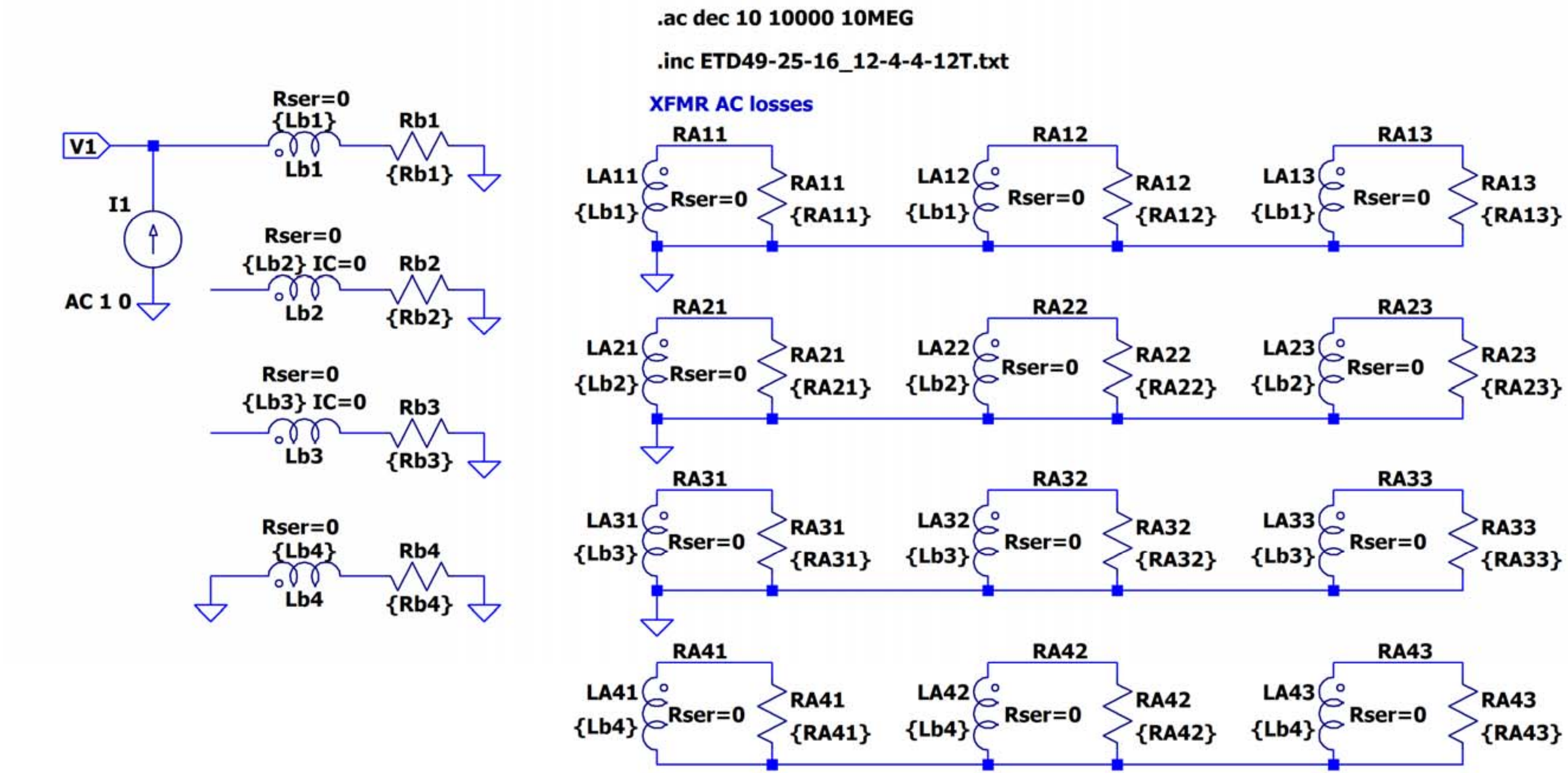


Self-Impedance SPICE Simulation



Self-Impedance SPICE Simulation

Leakage-Impedance SPICE Simulation



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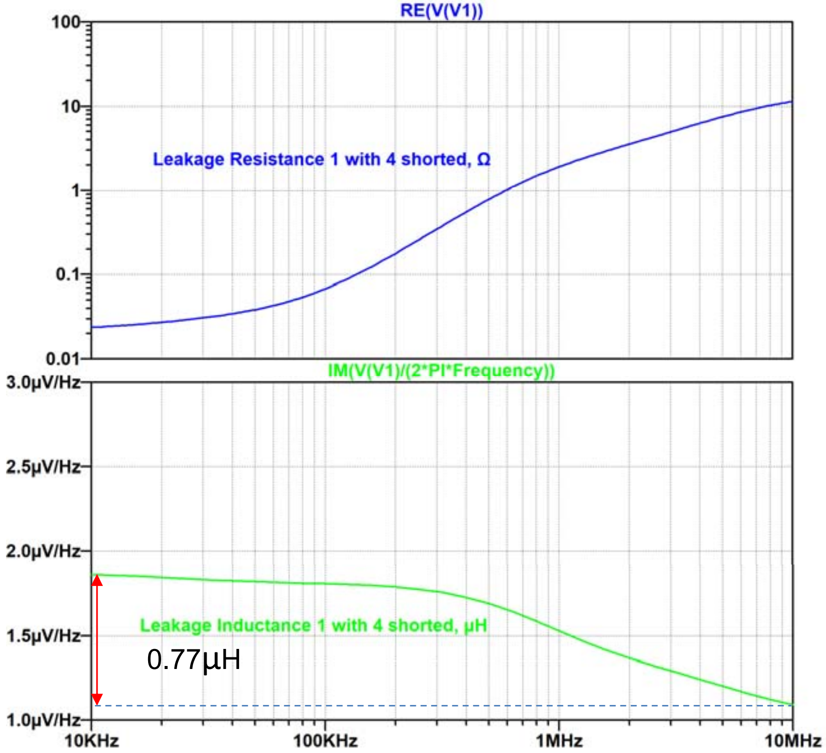
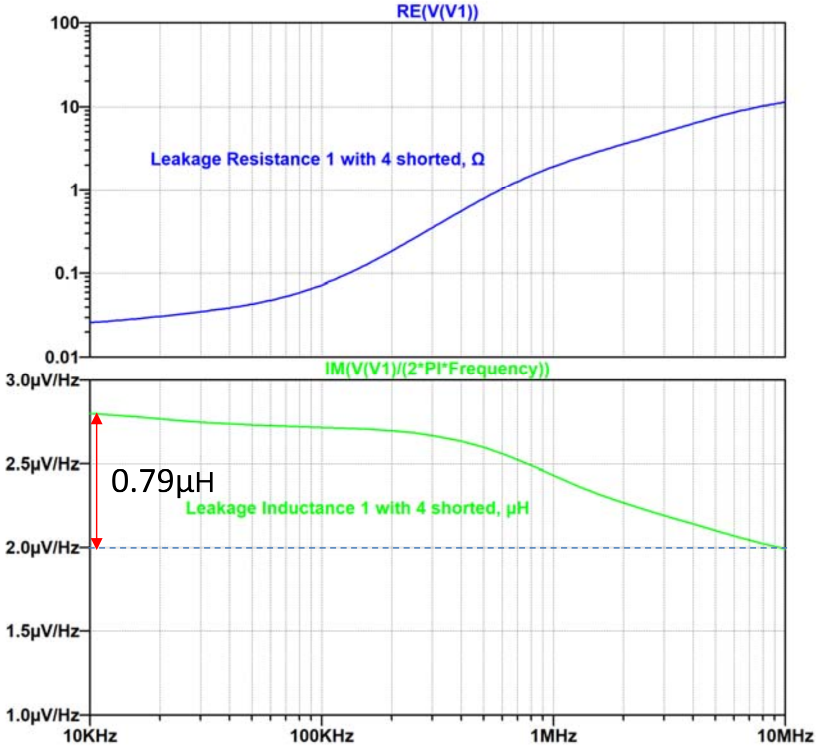
Leakage-Impedance SPICE Simulation

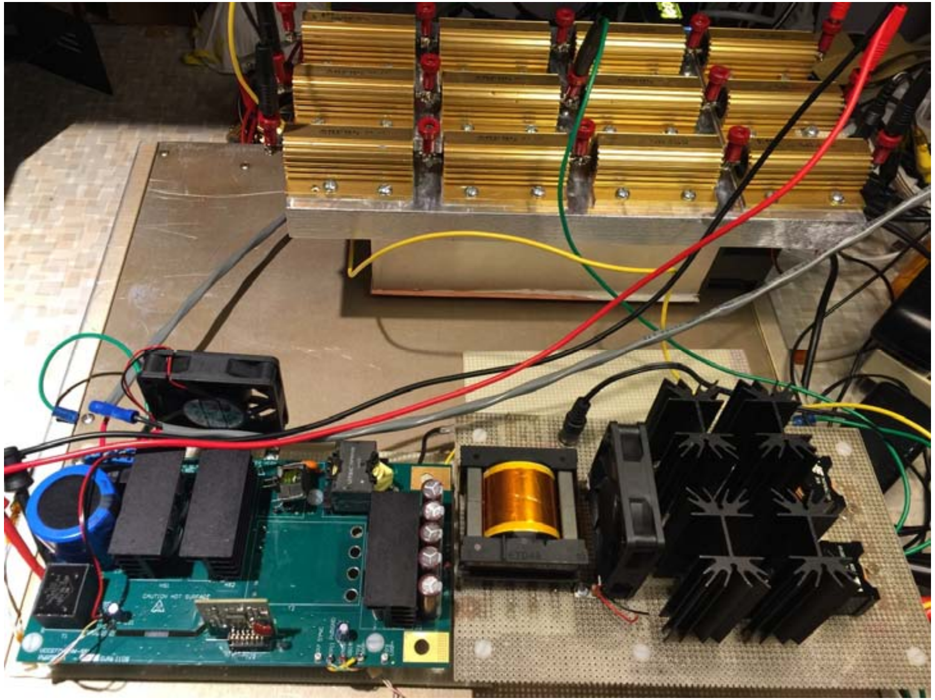
3 Layers 2 mil Nomex



10 Layers 2 mil Nomex

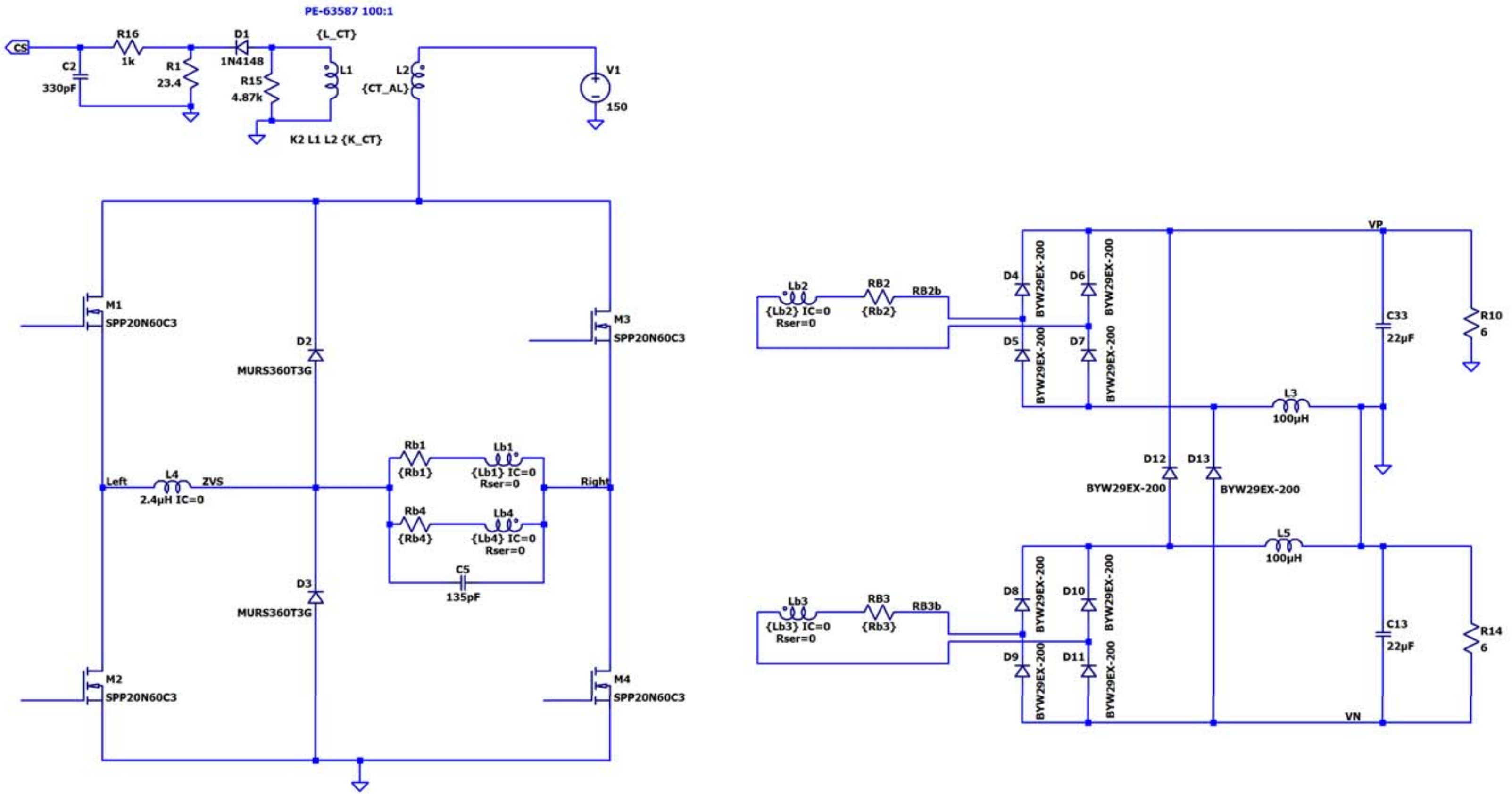


Phase-Shifted Bridge Converter



TI UCC27714EVM-551Demo Board with new transformer and output circuit

Phase-Shifted Bridge Converter Power Stage



Simulation and Measured Test Results, Lzvs = 2.4 μH SPICE based on FEA Data

2.4uH	Vin, V	Iin, A	Vpos	Vneg	Rpos	Rneg	Pin	Pout	Eff
Measured	149.8	2.207	29.97	29.97	6.009	5.997	330.6	299.3	90.5%
SPICE	149.8	2.258	29.97	29.96	6.009	5.997	338.2	299.2	88.4%
Measured	149.9	1.166	29.97	31.15	6.009	106.5	174.8	158.6	90.7%
SPICE	149.9	1.172	29.96	32.82	6.009	106.5	175.7	159.5	90.8%
Measured	150.1	0.1023	29.98	2.03	106.0	5.997	15.4	8.5	55.2%
SPICE	150.1	0.1210	29.96	2.03	106.0	5.997	18.2	8.5	46.6%
Measured	150.1	0.1336	29.97	30.18	106.0	106.5	20.1	8.6	42.6%
SPICE	150.1	0.1379	29.91	29.92	106.0	106.5	20.7	8.4	40.6%

Utah State University ASPIRE NSF Engineering Research Center

Simulation and Measured Test Results, Lzvs = 2.4 μH SPICE based on Measured Data

2.4uH	Vin, V	Iin, A	Vpos	Vneg	Rpos	Rneg	Pin	Pout	Eff
Measured	149.8	2.207	29.97	29.97	6.009	5.997	330.6	299.3	90.5%
SPICE	149.8	2.260	29.97	29.94	6.009	5.997	338.5	298.9	88.3%
Measured	149.9	1.166	29.97	31.15	6.009	106.5	174.8	158.6	90.7%
SPICE	149.9	1.176	29.96	32.60	6.009	106.5	176.3	159.4	90.4%
Measured	150.1	0.1023	29.98	2.03	106.0	5.997	15.4	8.5	55.2%
SPICE	150.1	0.1200	29.96	1.99	106.0	5.997	18.0	8.5	47.0%
Measured	150.1	0.1336	29.97	30.18	106.0	106.5	20.1	8.6	42.6%
SPICE	150.1	0.1152	29.96	29.97	106.0	106.5	17.3	8.4	48.8%

Simulation and Measured Test Results, Lzvs = 10.7 µH SPICE based on FEA Data

10.7uH	Vin, V	Iin, A	Vpos	Vneg	Rpos	Rneg	Pin	Pout	Eff
Measured	149.6	2.218	29.97	29.99	6.009	5.997	331.8	299.5	90.2%
SPICE	149.6	2.269	29.97	29.96	6.009	5.997	339.4	299.2	88.2%
Measured	149.9	1.170	29.97	31.03	6.009	106.5	175.4	158.5	90.4%
SPICE	149.9	1.168	29.96	31.05	6.009	106.5	175.1	158.4	90.5%
Measured	150.1	0.0897	29.98	2.09	106.0	5.997	13.5	8.5	63.0%
SPICE	150.1	0.10761	29.96	2.14	106.0	5.997	16.2	8.5	52.4%
Measured	150.1	0.1377	29.98	30.03	106.0	106.5	20.7	8.5	41.0%
SPICE	150.1	0.1141	29.94	29.95	106.0	106.5	17.1	8.4	49.1%

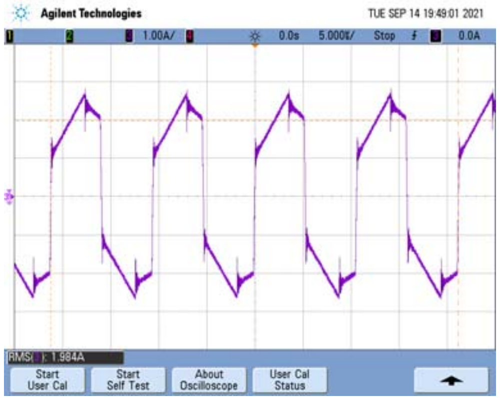
Simulation and Measured Test Results, Lzvs = 10.7 µH SPICE based on Measured Data

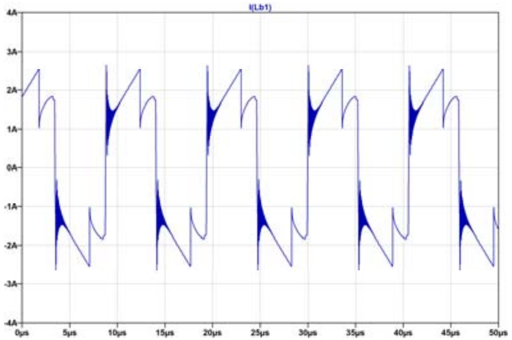
10.7uH	Vin, V	Iin, A	Vpos	Vneg	Rpos	Rneg	Pin	Pout	Eff
Measured	149.6	2.218	29.97	29.99	6.009	5.997	331.8	299.5	90.2%
SPICE	149.6	2.272	29.97	29.94	6.009	5.997	339.9	299.0	87.9%
Measured	149.9	1.170	29.97	31.03	6.009	106.5	175.4	158.5	90.4%
SPICE	149.9	1.195	29.96	30.68	6.009	106.5	179.2	158.3	88.3%
Measured	150.1	0.0897	29.98	2.09	106.0	5.997	13.5	8.5	63.0%
SPICE	150.1	0.1075	29.97	2.05	106.0	5.997	16.1	8.5	52.5%
Measured	150.1	0.1377	29.98	30.03	106.0	106.5	20.7	8.5	41.0%
SPICE	150.1	0.1148	29.95	29.97	106.0	106.5	17.2	8.4	49.0%

Transformer Primary Currents Lzvs = 2.4 μH, FEA-Based SPICE

Layer 1

1.984A RMS





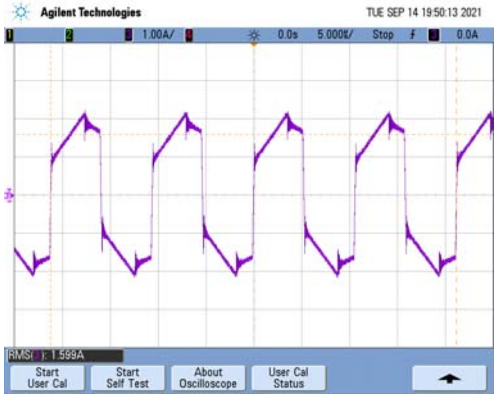
1.819A RMS, -8% error

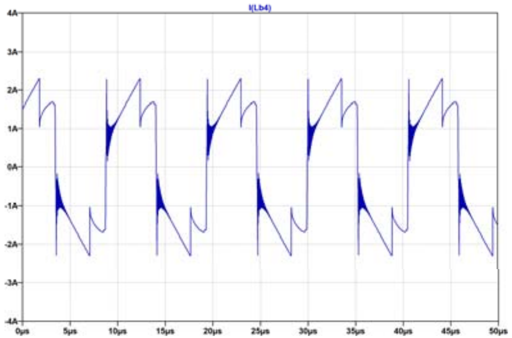
Ratio: 1.24

Ratio: 1.15, -7% error

Layer 4

1.599A RMS



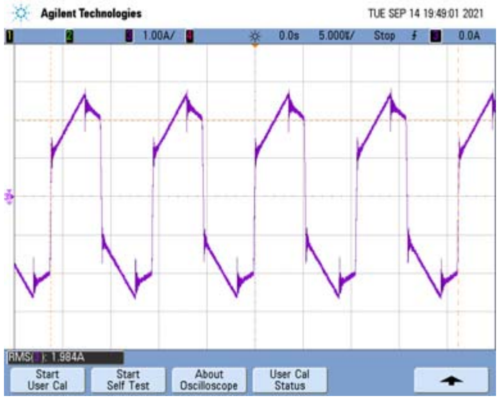


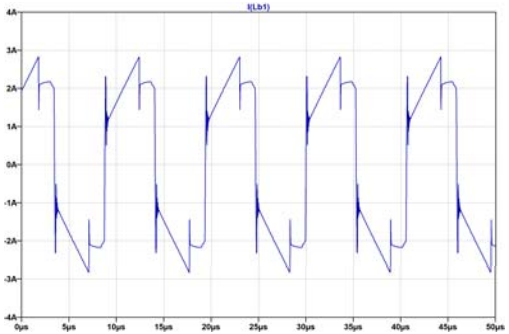
1.580A RMS. -1% error

Transformer Primary Currents Lzvs = 2.4 µH, Meas-Based SPICE

Layer 1

1.984A RMS



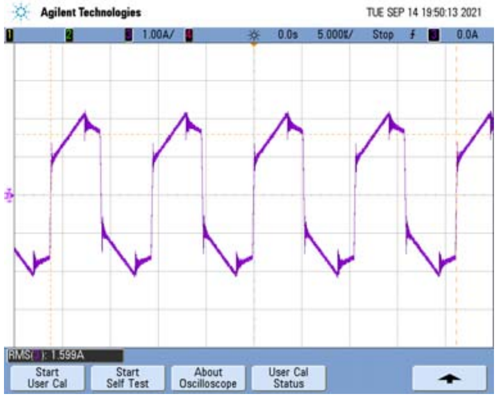


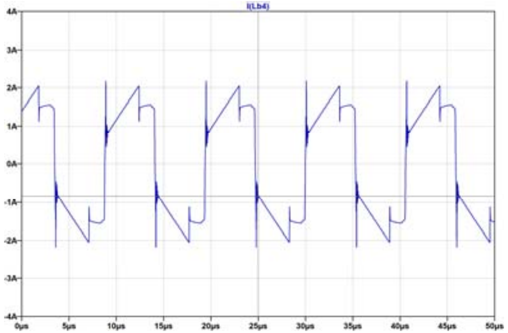
2.039A RMS, +3% error

Ratio: 1.24

Ratio: 1.39, +12% error

Layer 4





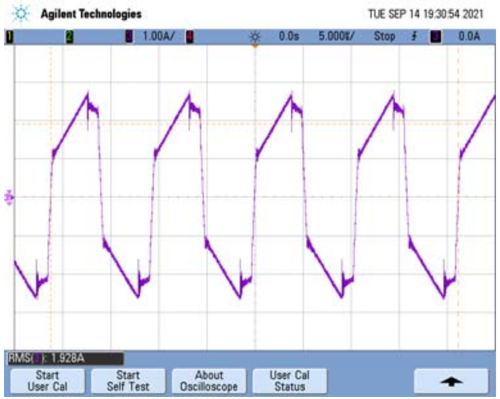
1.599A RMS

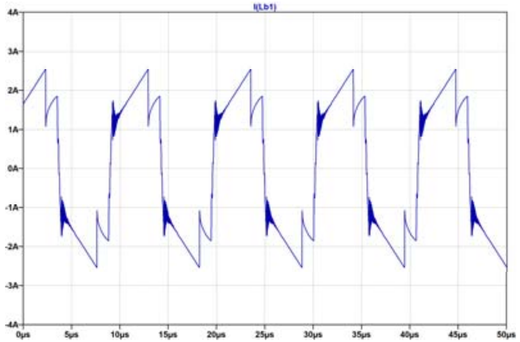
1.463A RMS. -8% error

Transformer Primary Currents Lzvs = 10.7 µH, FEA-Based SPICE

Layer 1

1.928A RMS





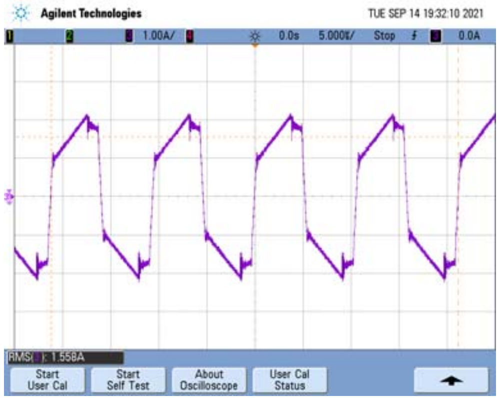
1.770A RMS, -8% error

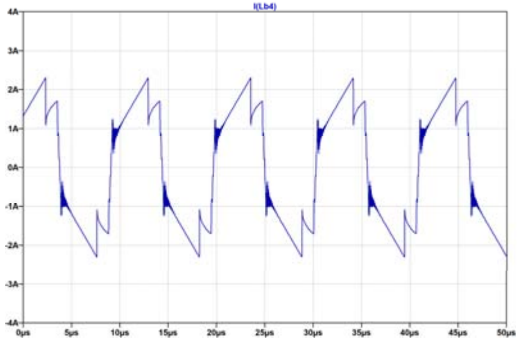
Ratio: 1.24

Ratio: 1.13, -6% error

Layer 4

1.558A RMS



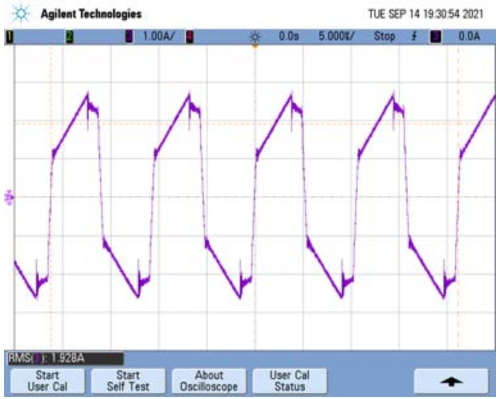


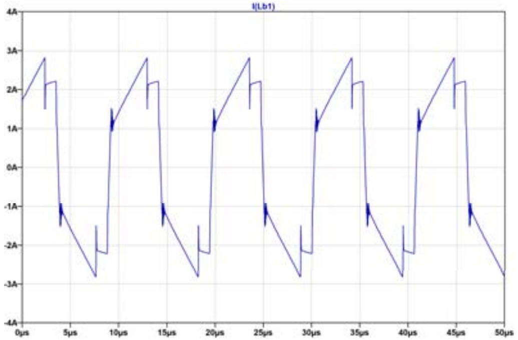
1.527A RMS. -2% error

Transformer Primary Currents Lzvs = 10.7 µH, Meas-Based SPICE

Layer 1

1.928A RMS





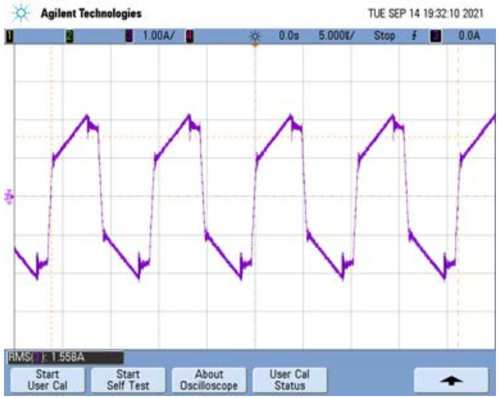
1.987A RMS, +3% error

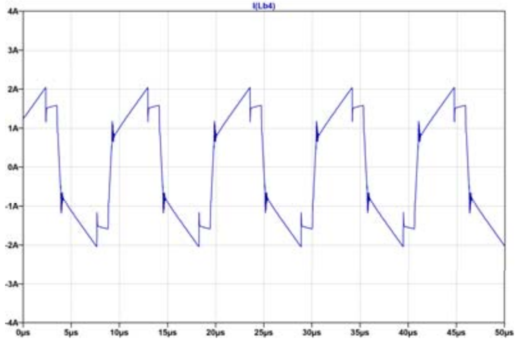
Ratio: 1.24

Ratio: 1.39, +12% error

Layer 4

1.558A RMS





1.428A, RMS. -8% error

Phase-Shifted Bridge Transformer Loss Waveforms, FEA

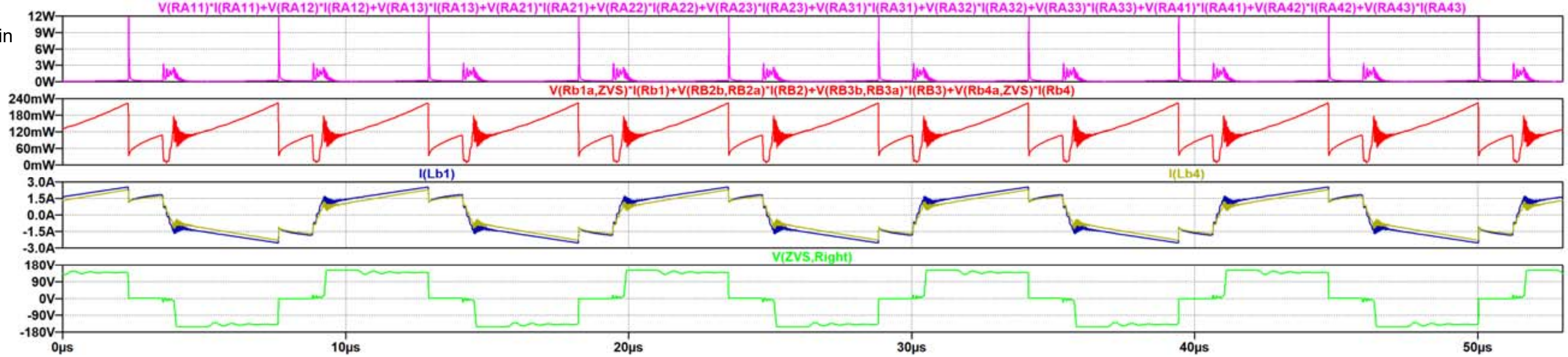
10.7 µH ZVS Inductor

High Frequency losses in Auxiliary Resistors
328.58mW

Losses in DC Resistors
128.1mW

Primary Currents

Primary Voltage



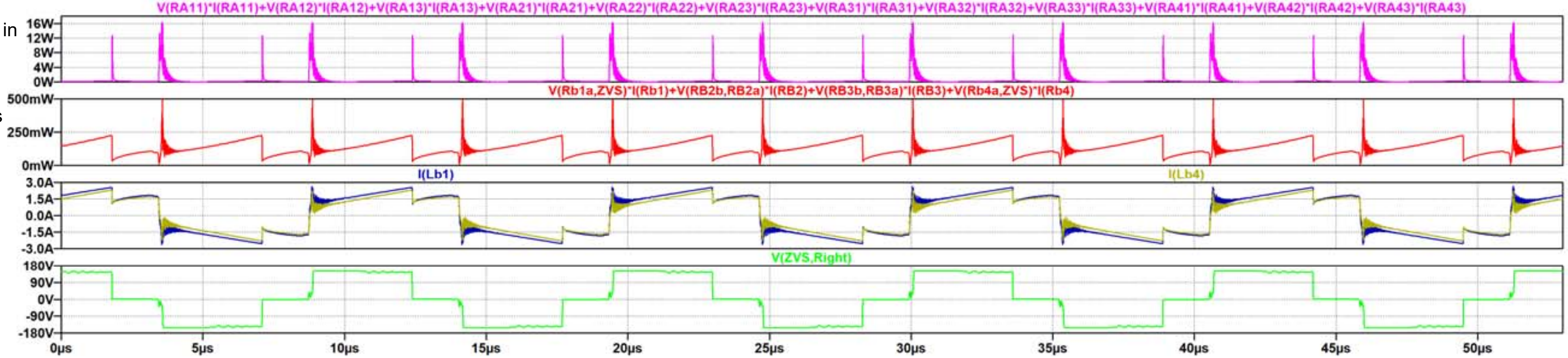
2.4 µH ZVS Inductor

High Frequency losses in Auxiliary Resistors
599.09mW

Losses in DC Resistors
132.3mW

Primary Currents

Primary Voltage



Phase-Shifted Bridge Transformer Loss Waveforms, Meas

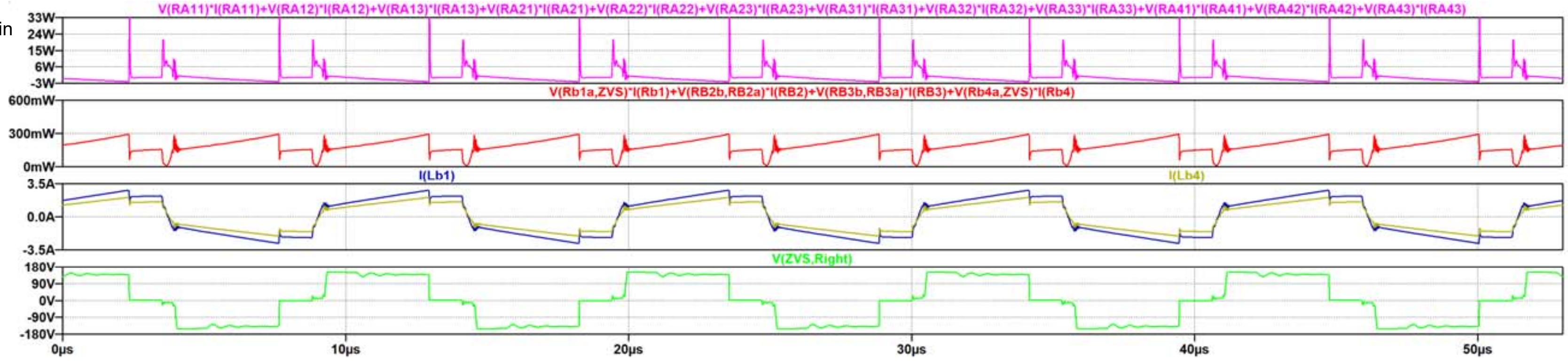
10.7 μH ZVS Inductor

High Frequency losses in Auxiliary Resistors
476.87mW

Losses in DC Resistors
189.5mW

Primary Currents

Primary Voltage



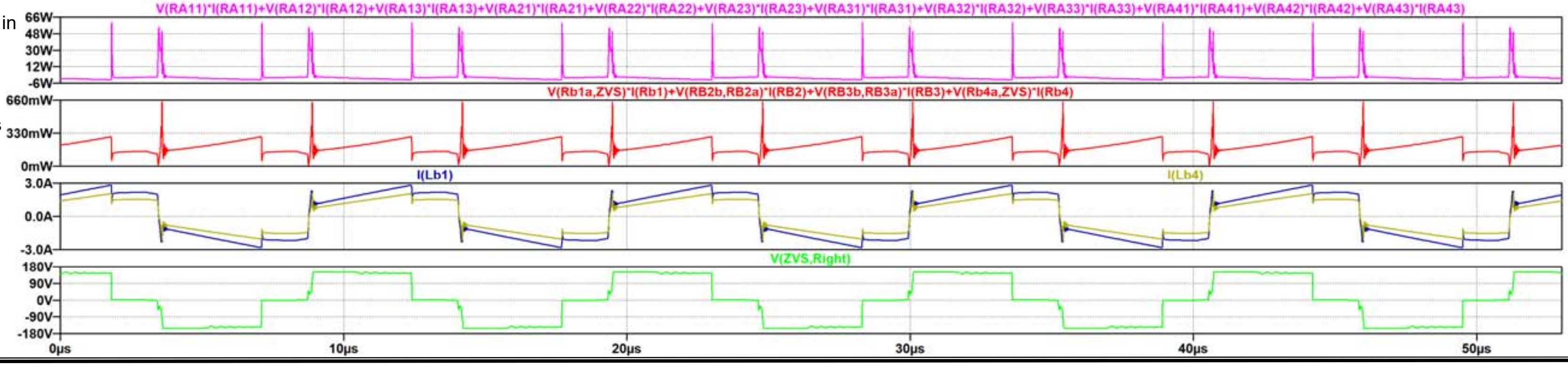
2.4 μH ZVS Inductor

High Frequency losses in Auxiliary Resistors
988.19mW

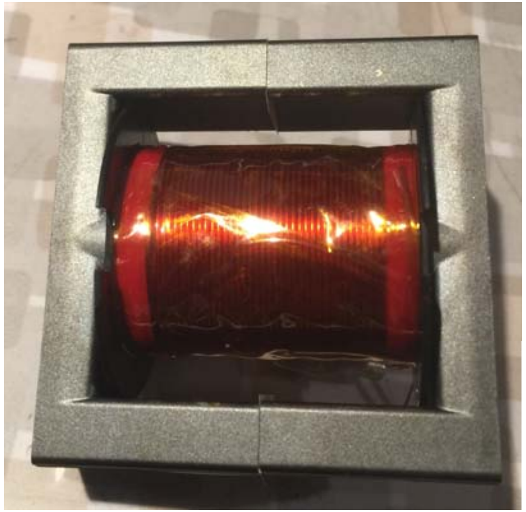
Losses in DC Resistors
192.43mW

Primary Currents

Primary Voltage



Maxwell 2D Pulse Test Transformer Model



All wires 22 AWG

All windings 38 turns

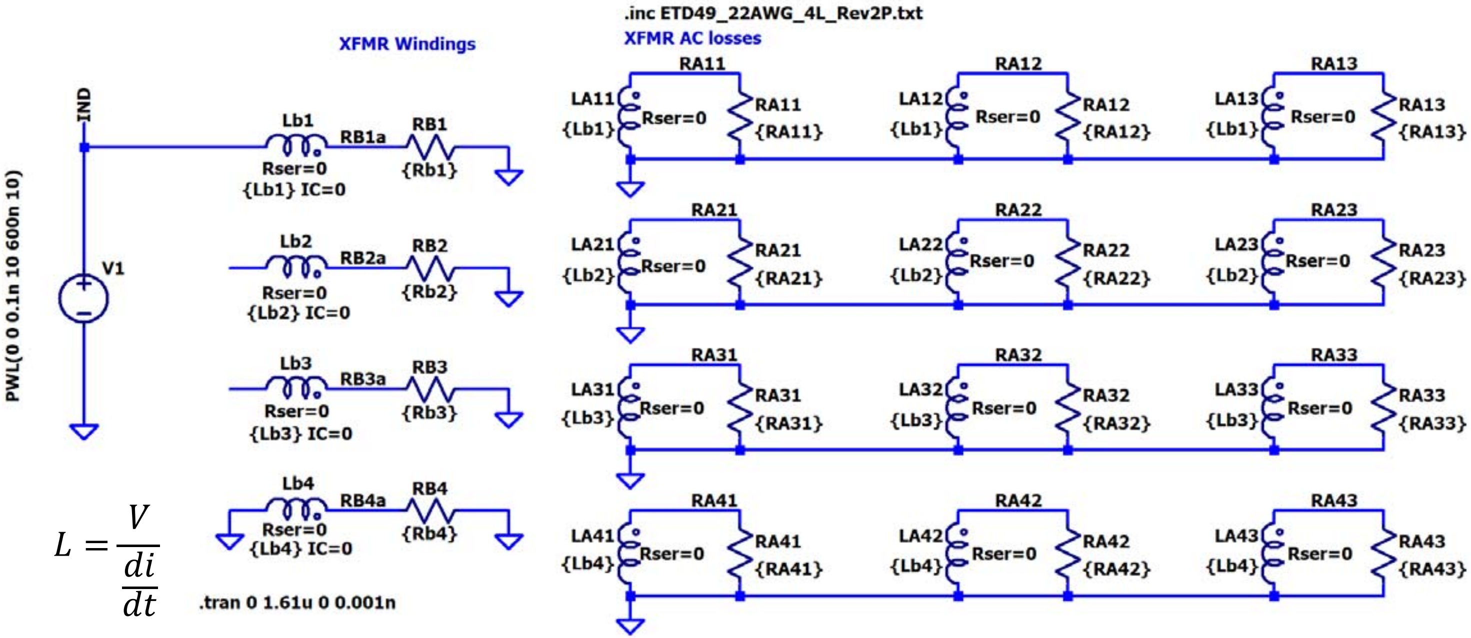
2 layers 2 mil tape between layers, 0.1mm

Core: ETD49/25/16-3C97

Gap: 3 mil spacer

Bobbin: TDK B66368B1020T001

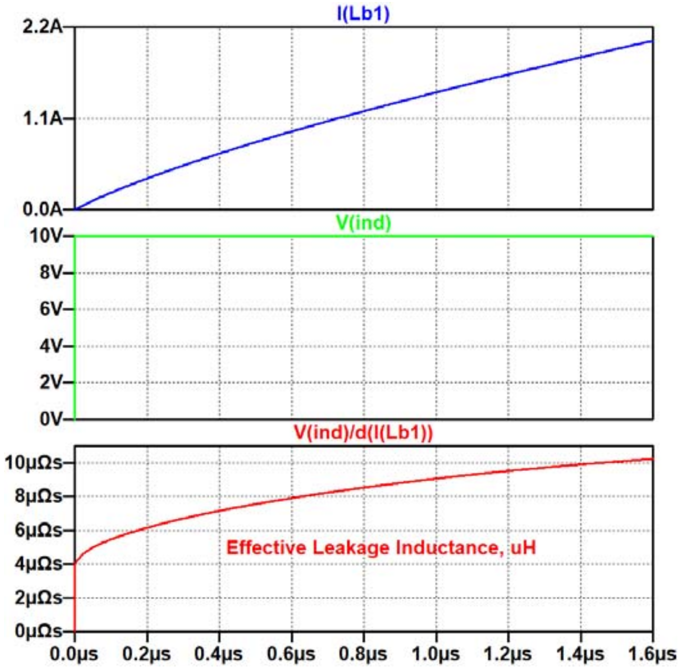
Leakage Inductance Variation with Pulse Width

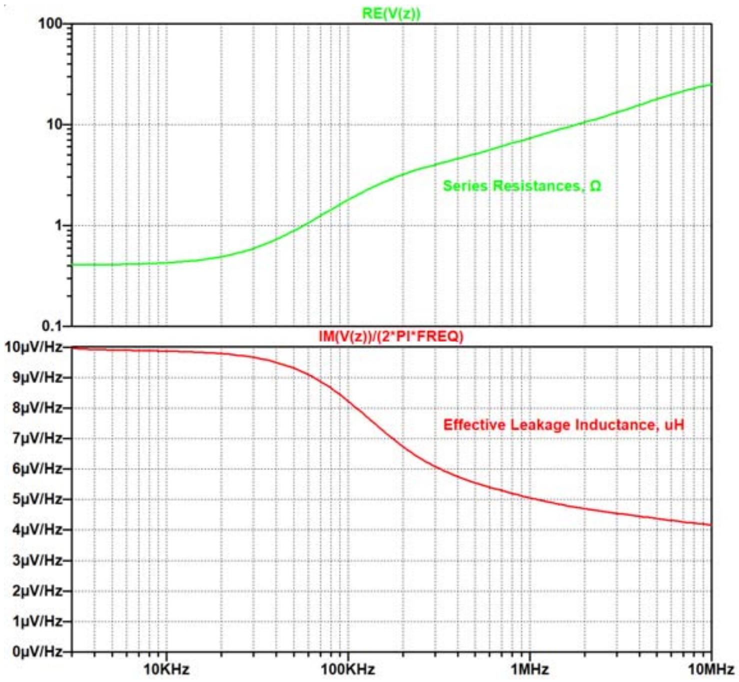


$$V = L\frac{di}{dt} \qquad L = \frac{V}{\frac{di}{dt}}$$

A voltage pulse is applied to winding 1 while winding 4 is shorted in order to show how the apparent leakage inductance varies with time.

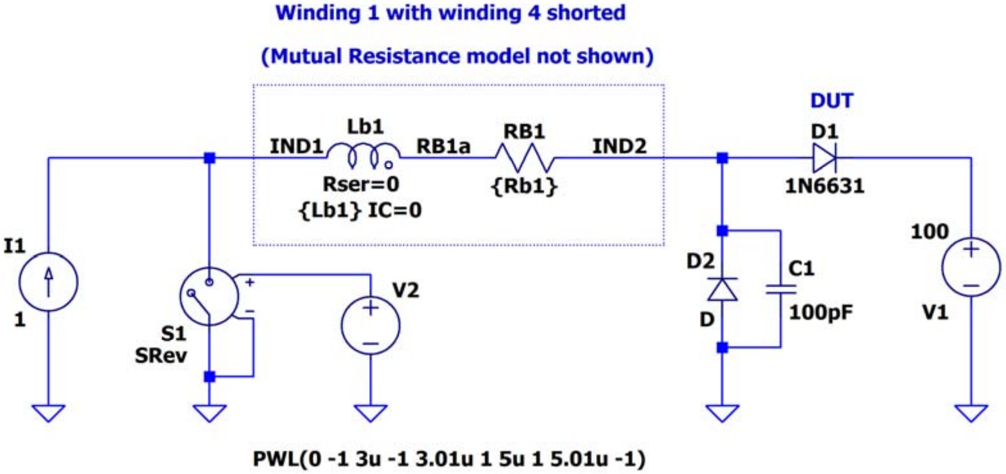
Leakage Inductance Simulations





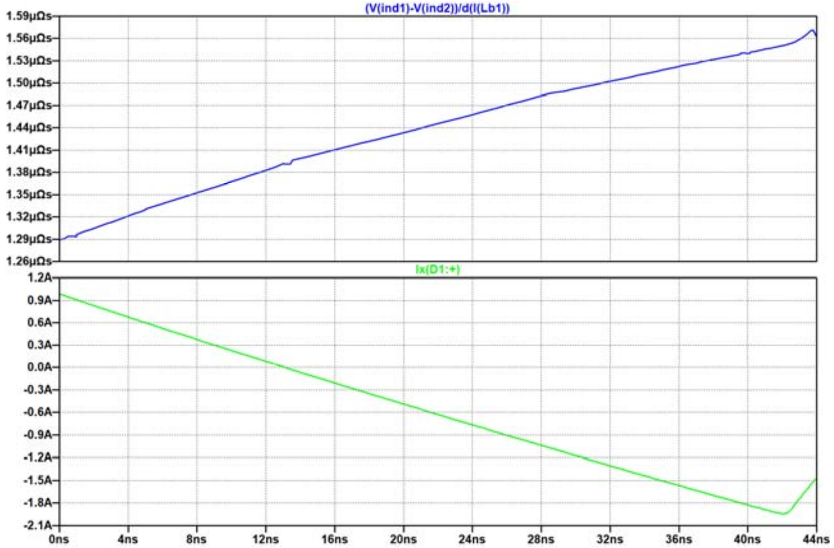
- The range for the effective leakage inductance in the time domain is close to the range in the frequency domain

Diode Reverse Recovery Test Circuit [13]

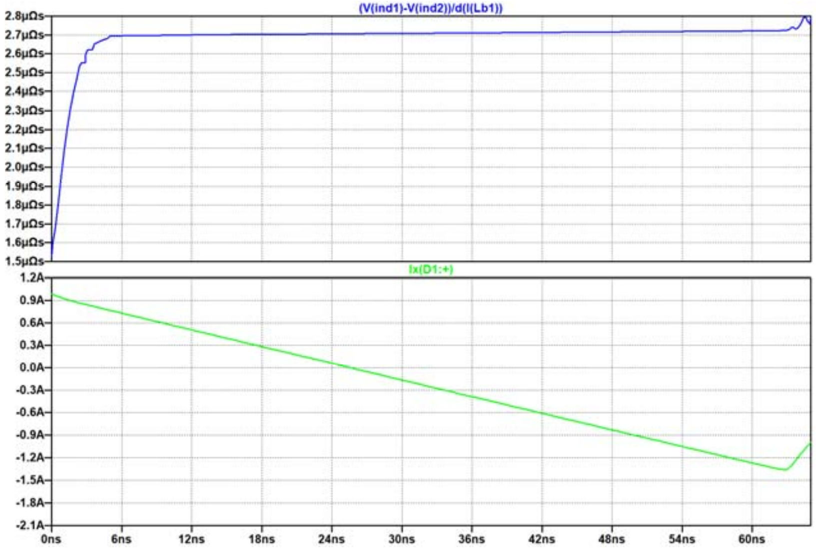



```
.SUBCKT 1N6631 anode cathode
.param IS=300n N=1.5 Tau=60n Y0=1 alpha=5MEG va=0.1
Dpn anode N001 Dj
.MODEL Dj D( IS {IS} N {N} )
Cdiff N002 diff 1
Vdiff diff 0 0V
Ediff  N003 0 Value {I(Vdiff)}
Hcsense N005 0 VHcsense 1
VHcsense N001 N004 0V
G_charge_current anode N004 N003 0 1
E_charge_calculator N002 0 VALUE{Tau*(V(N005)-va*V(N003))}
G_base_region_current N004 cathode VALUE {(V(N004) - V(cathode))*(Y0+alpha*V(N002)) }
.ENDS
```

Diode Reverse Recovery Comparison



Mutual Resistance Model Enabled



Mutual Resistance Model Disabled

- The mutual resistance model reduces the effective inductance and increases the peak reverse recovery current

Conclusions

- This mutual impedance circuit model can be made to match FEA results very closely
- The mutual impedance circuit model matches measured data fairly well, but the range of frequencies where accurate results can be obtained are more limited compared to FEA-based models because of capacitive effects
- Compensating for the winding capacitances can extend the frequency range of accurate measurements
- The SPICE circuit models produced from FEA simulations are more accurate at predicting the current sharing than the models produced by LCR measurements
- The SPICE circuit models produced from both the FEA simulations and LCR measurements accurately predict the dc measurements
- All of the model variations were able to capture a loading condition of concern

Conclusions (continued)

- The leakage inductance for closely-coupled winding pairs decreases with frequency
- The inductance decrease is due to skin and proximity effects
- The effective leakage inductance for pulsed waveforms can be determined by dividing the applied voltage by the time derivative of the current
- The effective leakage inductance for short pulses is less than for longer pulses
- The currents produced by the reverse recovery of fast diodes connected to transformer outputs depend on the high-frequency leakage inductance values
- Example files can be found at http://www.verimod.com/resources.html

References

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