Derivation and Real World Use of DQ Transform for Motor Drives

Presented to

IEEE Power Electronics Society
San Francisco Bay Area Chapter

By

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22 February 2018

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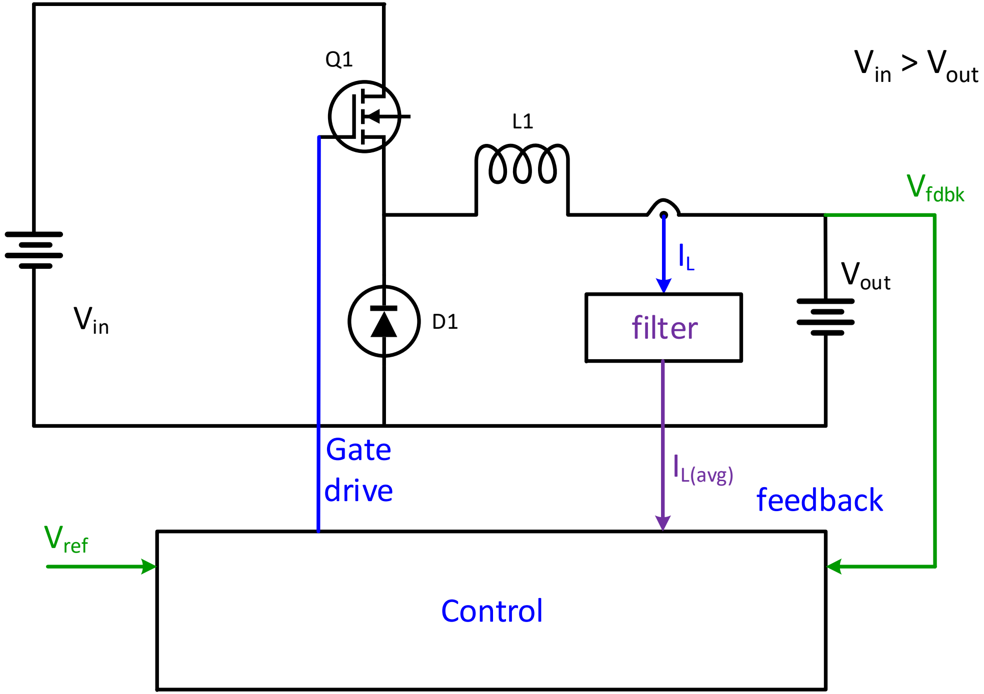
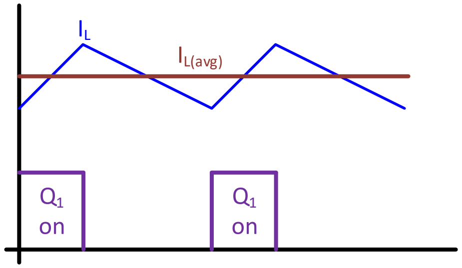
Agenda

- 1. Control of DC current
- 2. Control of AC current (no DQ used)
- 3. Motivation for DQ current control in 3 phase systems
- 4. DQ motor model and speed control
- 5. Issues with DQ control
- 6. Induction Motor

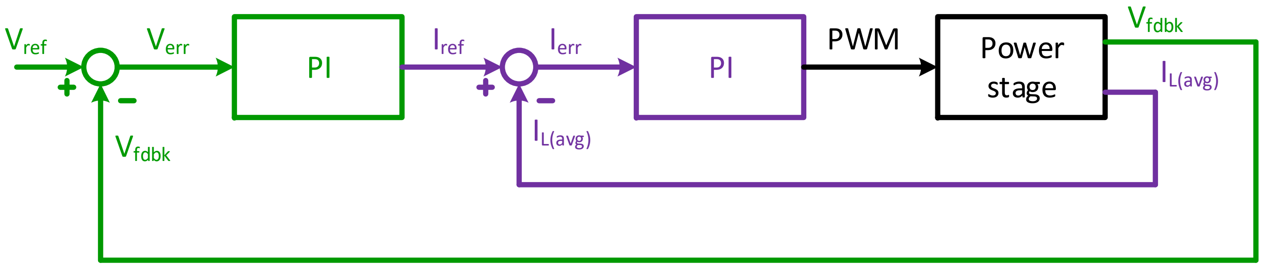
1. Control of DC current in Buck Power Supply





Control of DC Current in Buck Power Supply

- Inductor current is continuous
  - Many different ways to obtain current measurement, literature full of examples
- Set point of inner current loop determined by outer voltage loop



- Control loop can be implemented using analog or digital techniques
  - Standard control problem

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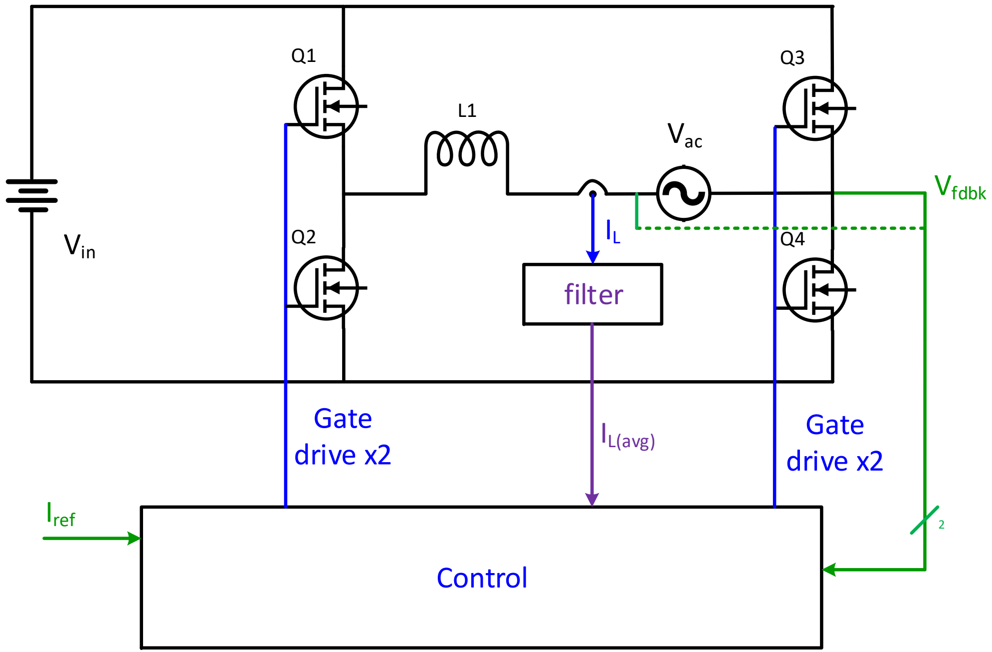
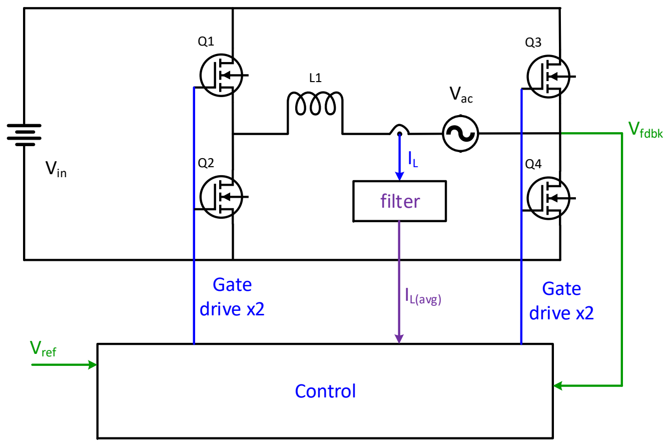
AC Current Control

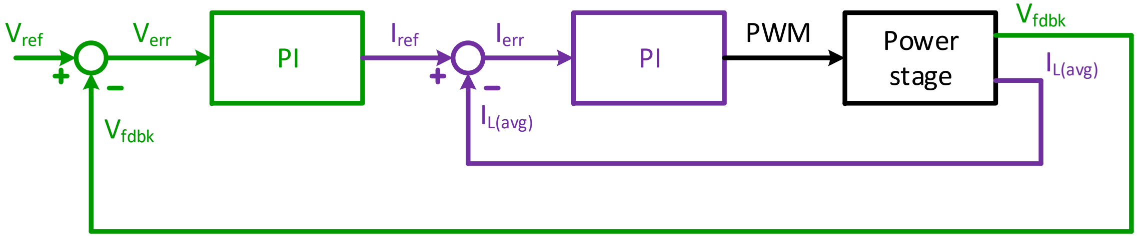
- Replace dc load with ac load
  - Could be resistor, motor back emf or ac source
- Bipolar voltage applied across inductor
- Current in phase with V_{ac}
- Switch operation
  - $Q_{1,4}$ PWM'ed for 50% of the time
  - $Q_{2,3}$ off during this time
  - Roles switch for the other 50% of time



Issues with AC Current Control in Inductor

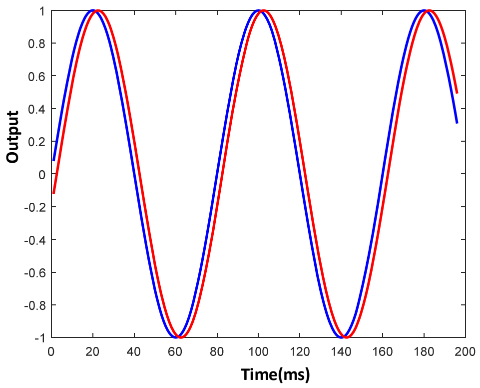
- Bandwidth of current feedback filter does not introduce phase lag in feedback current
  - $f_{PWM} >> f_{ac}$
- All currents and voltages are now ac
- Bandwidth of PI loop introduces a phase lag between I_{ref} and $I_{L(avg)}$
  - Lag increases as f_{ac} increases

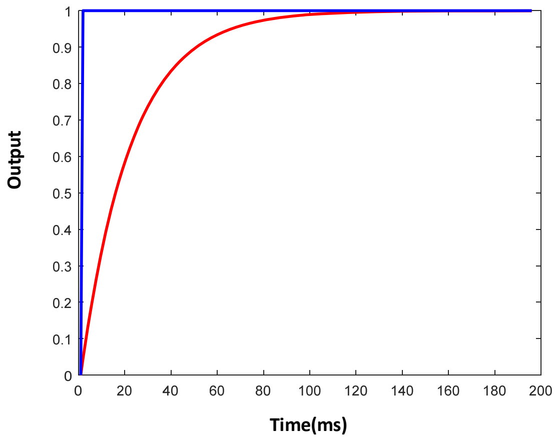


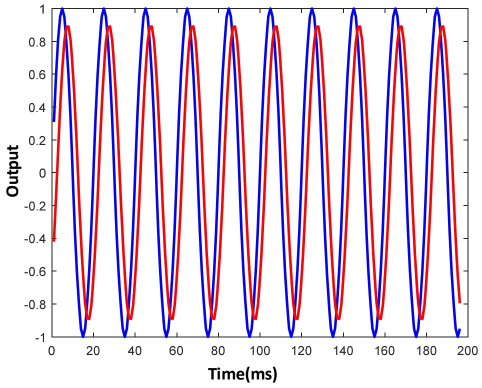


Compare Response of Both Circuits

- DC circuit attains steady state with no dc error
- AC circuit always shows a lag
  - Increases with frequency
  - Amplitude decrease also

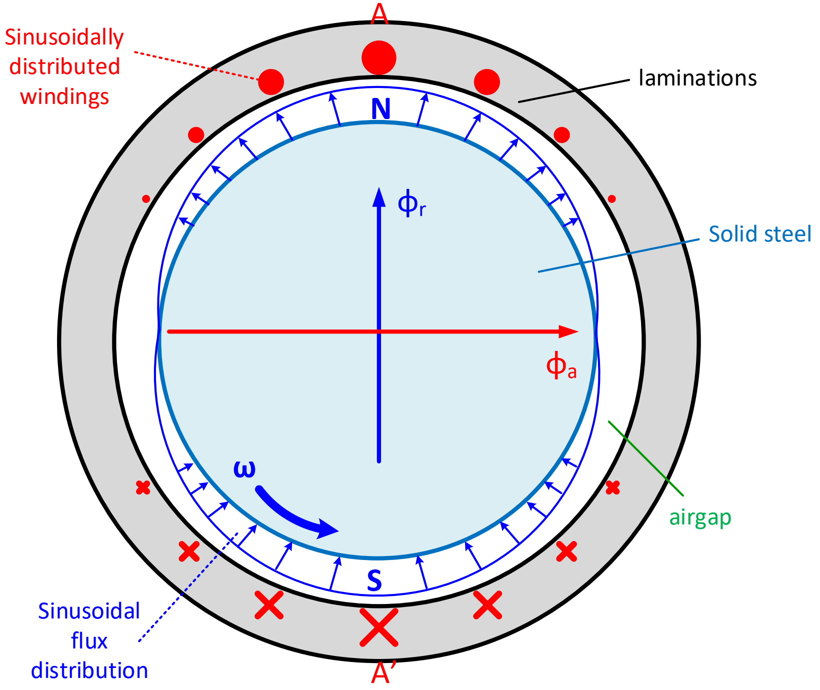






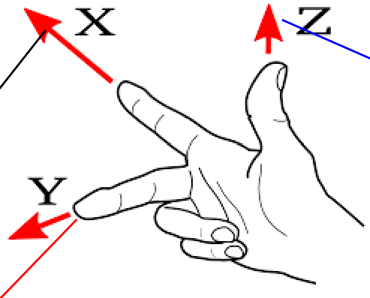
Surface Mounted Permanent Magnet Motor

- Shows phase A only (for clarity)
  - B, C are displaced by 120°
- At this angle, the rotor flux (ϕ_r) does not link coil AA'
  - Rate of change of flux linkage ($\frac{d\lambda}{dt}$) is a maximum however
  - λ : flux linking coil = N ϕ_r
  - N : number of turns in coil A



Motor Torque Direction

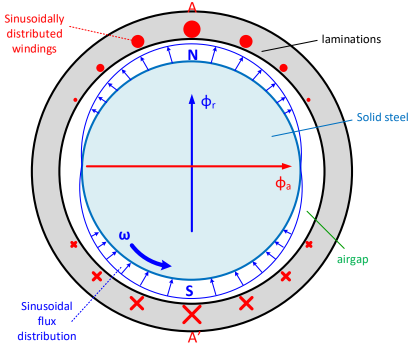
- Torque direction
  - Use right hand rule
  - $T = \boldsymbol{I} \ \boldsymbol{x} \ \boldsymbol{B} \quad {}^{N}/_{m^2}$
  - $\boldsymbol{I}$, $\boldsymbol{B}$ are vectors
  - Point A : $\boldsymbol{I}$ out of the page
  - Point A : $\boldsymbol{I}'$ into the page
    - T is CCW



rotor magnetic field (vertical)

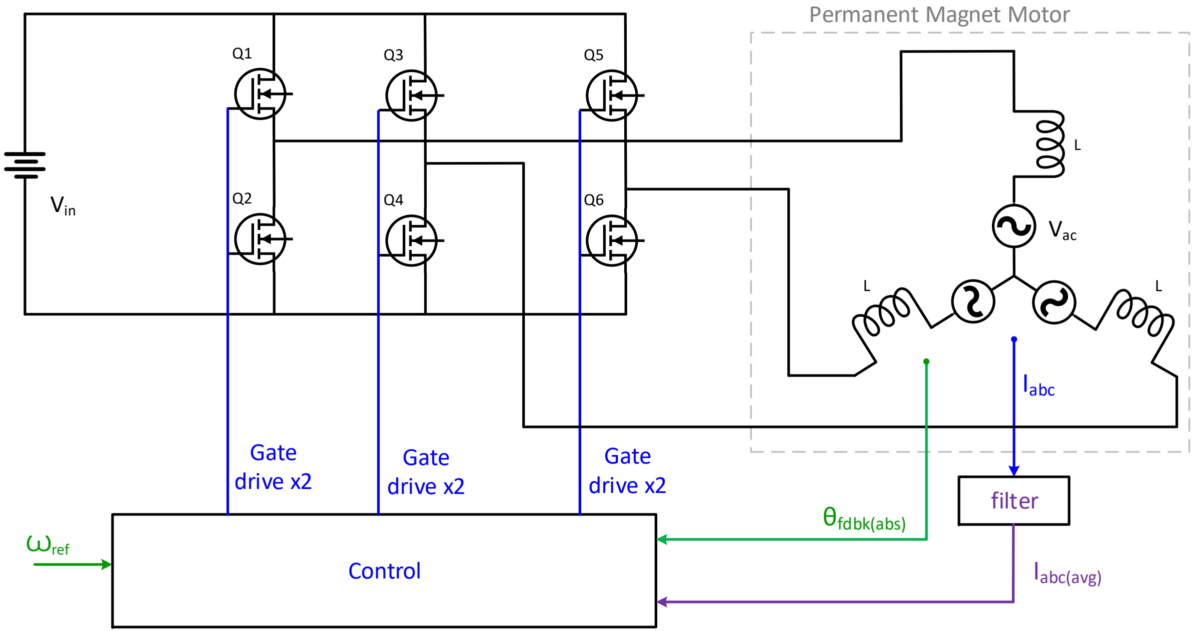
Force (tangential CCW at point A)

current (out of the page)



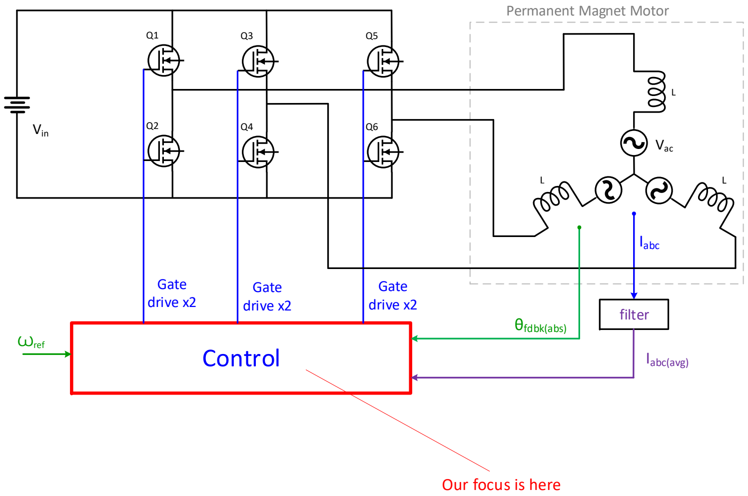
3 Phase Permanent Magnet Motor Speed Control (Surface Magnets)

- Control speed of motor
- Have current feedback and absolute position feedback
  - Feedback methods are outside present scope
  - Filter delay not significant
- Motor represents 3 phase balanced load



Current Control

- Will not look at PWM implementation
  - Assume commanded voltage is impressed on motor without any issues
- Close ac current loop using I_{abc} directly
  - Have same problem as single phase case
  - Phase lag and amplitude reduction as frequency increases
- To maximize torque/amp, want currents in phase with respective back emfs
  - Back emf referenced here is line-neutral, NOT line-line
    - Absolute position sensor needs to be aligned with line-neutral voltage [**]
    - [**] : source of confusion
- No field weakening



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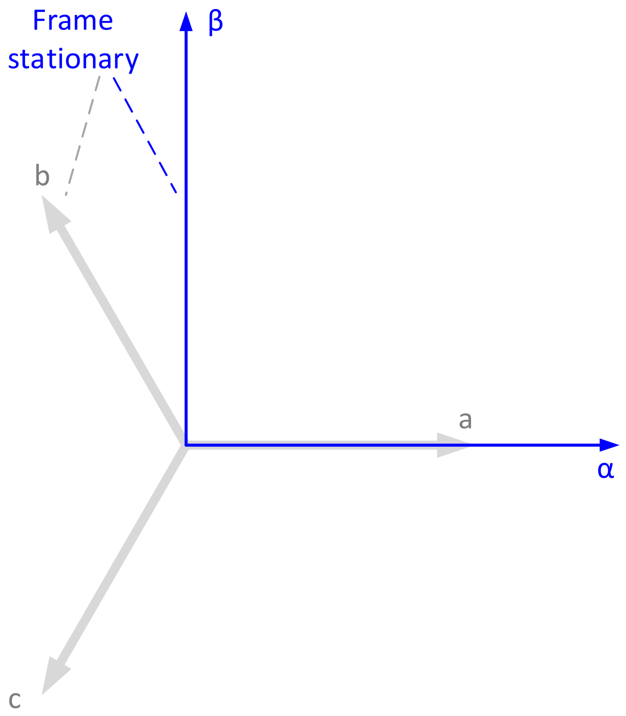
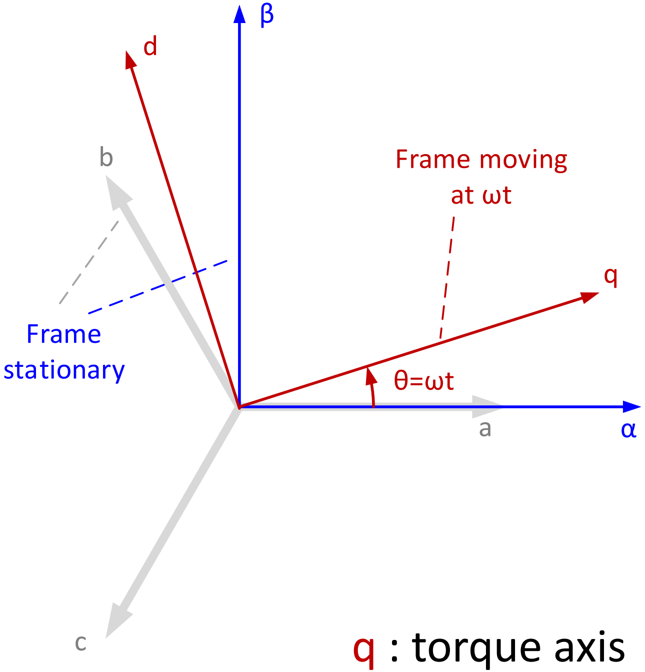
3 Phase to 2 Phase : Stationary Frame

- $V_{an} = V_0 \sin\theta$
- $V_{bn} = V_0 \sin(\theta - \frac{2}{3}pi)$
- $V_{cn} = V_0 \sin(\theta - \frac{4}{3}pi)$
  - $\theta = \omega t$
  - V_0 depends on speed and magnet strength
- Have 3 phase, balanced system
  - Can't control 3 current independently, only 2
  - Reduce from 3 phases to 2 : no vector addition
- Call these axes α, β (using European notation)
  - ***α*** aligned with ***a***
  - $V_{\alpha n} = V_{an} - \frac{1}{2}(V_{bn} + V_{cn}) = \frac{3}{2}V_{an}$
  - $V_{\beta n} = \frac{\sqrt{3}}{2}(V_{bn} - V_{cn})$



2 Phase : Stationary (αβ) to Synchronous (dq) .. Finally !

- $V_\alpha = (V_q \cos\theta - V_d \sin\theta)$
- $V_\beta = (V_q \sin\theta \quad V_d \cos\theta)$
  - *note* : dropped n subscript on $\alpha\beta$
    - n subscript not used for d, q
  - [**] V_d, V_q are <u>peak</u> values
    - Same as peak V_{an}, V_{bn}, V_{cn}



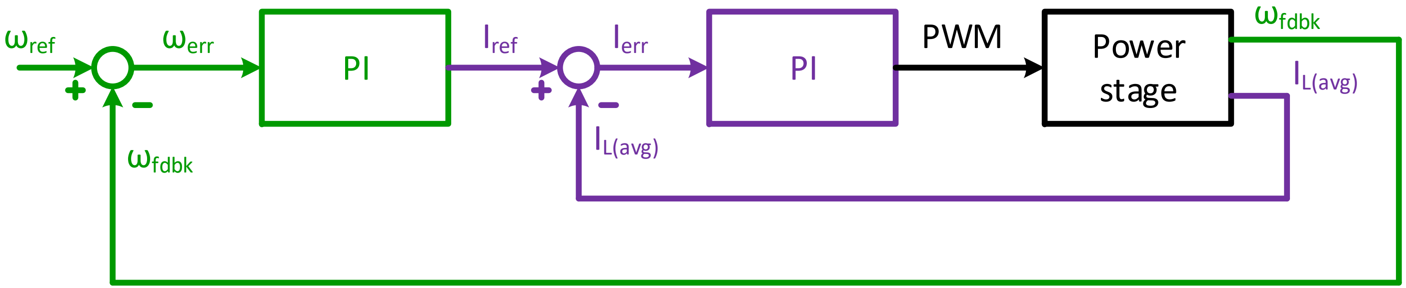
- $\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$

- Frame is stationary ➔ variables change with time
- Frame is synchronous ➔ variables constant with time

q : torque axis
d : flux axis
[**]

dq Current Loops

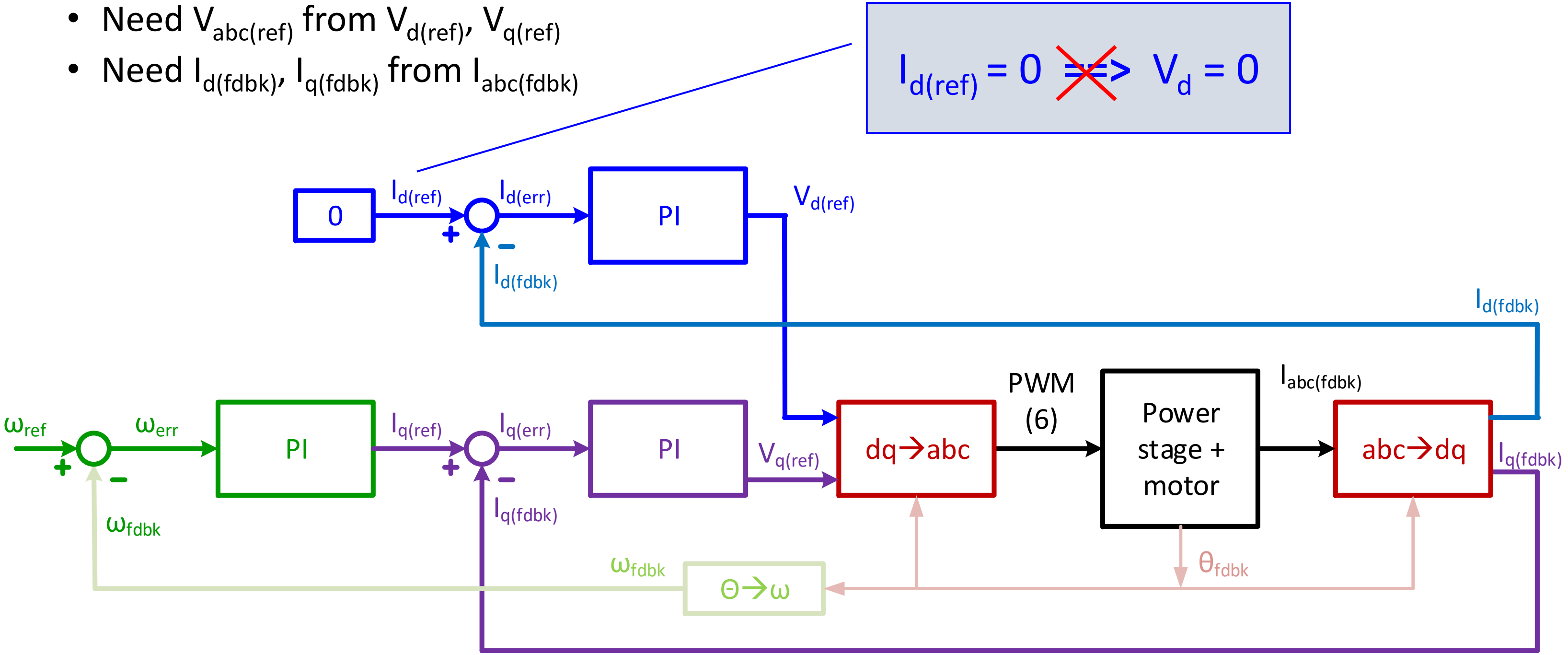
- Close current loops as before
  - Now have 2 closed loops : *d,q*
- Structure is the 'same' as dc current loop in buck
  - I_{ref}, $I_{L(avg)}$, I_{err} are vectors
  - Need more detail



- Outer loop is speed

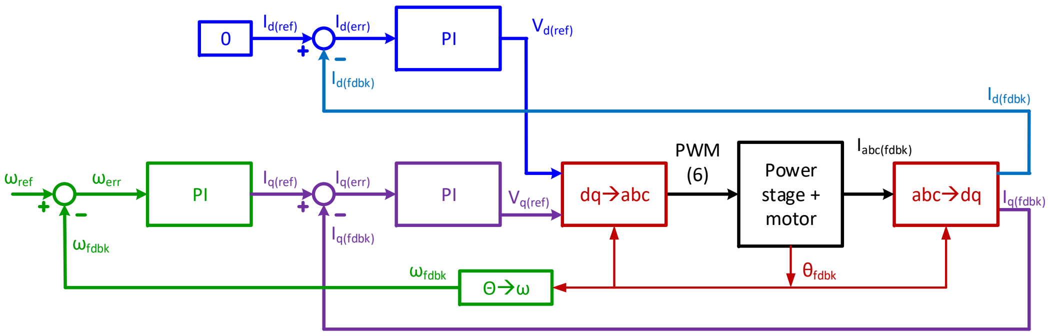
dq Current Loops In More Detail

- Now have 2 current loops
  - Need $V_{abc(ref)}$ from $V_{d(ref)}$, $V_{q(ref)}$
  - Need $I_{d(fdbk)}$, $I_{q(fdbk)}$ from $I_{abc(fdbk)}$



Details of *dq* → *abc* Command and *abc* → *dq* Feedback

- $\begin{bmatrix} I_q \\ I_d \end{bmatrix} = \frac{2}{3}\begin{bmatrix} cos\theta & \frac{\sqrt{3}}{2}sin\theta & -\frac{\sqrt{3}}{2}sin\theta \\ -sin\theta & \frac{\sqrt{3}}{2}cos\theta & -\frac{\sqrt{3}}{2}cos\theta \end{bmatrix}\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$
- $\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ \frac{1}{\sqrt{3}}sin\theta & \frac{1}{\sqrt{3}}cos\theta \\ \frac{-1}{\sqrt{3}}sin\theta & \frac{-1}{\sqrt{3}}cos\theta \end{bmatrix}\begin{bmatrix} V_q \\ V_d \end{bmatrix}$



Other Axes Transformations

- Presented one way of representing *dq* / *abc* transform here
- 2 phase notation
  - Stationary
  - Europe : αβ
  - US : $d^s q^s$
    - s : stator
  - Synchronous
  - Europe : dq
  - US : $d^e q^e$
    - e : excitation
- *dq* orientation
  - Krause, Bose, Lipo : d opposite direction
  - DeDoncker, Mohan : swap d, q
  - Fitzgerald : same

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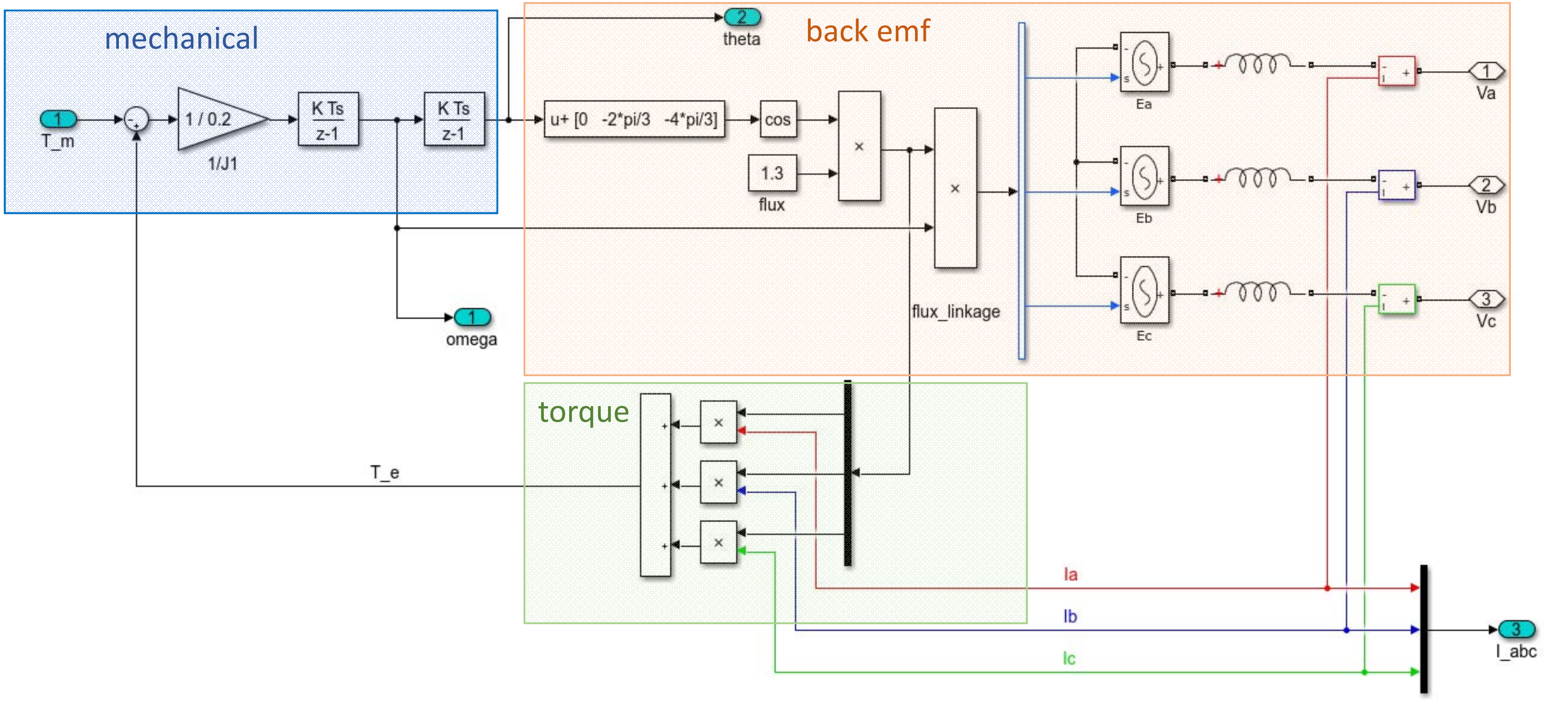
Simulink Model

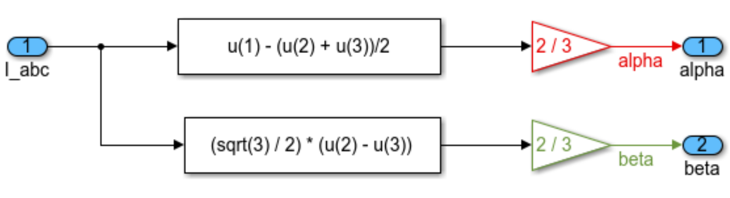
- Permanent Magnet Motor with Surface Mounted Magnets
  - modeled as three sinusoidal sources in series with inductance
  - Inductance is not a function of angle
  - Easier to match scope plots with simulation output if abc frame is used
    - Many models with pre-built motor blocks represent the motor in the dq frame, not the abc
- Model motor as three-phase ac source with series inductance
  - Load is a fan : $T_{load} \propto \omega^2$
- Drive motor with ideal sinusoidal source
  - Represents a PWM source whose frequency >> ac waveform
- No PWM used
  - Assume PWM effects are negligible
  - Valid if $f_{elec} << f_{PWM}$

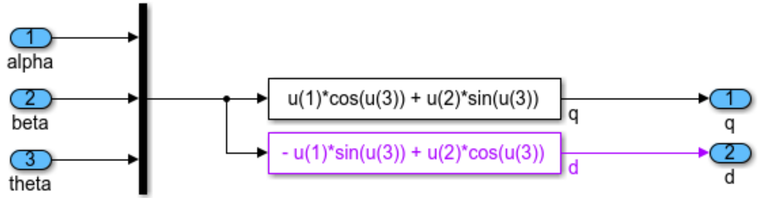
Motor

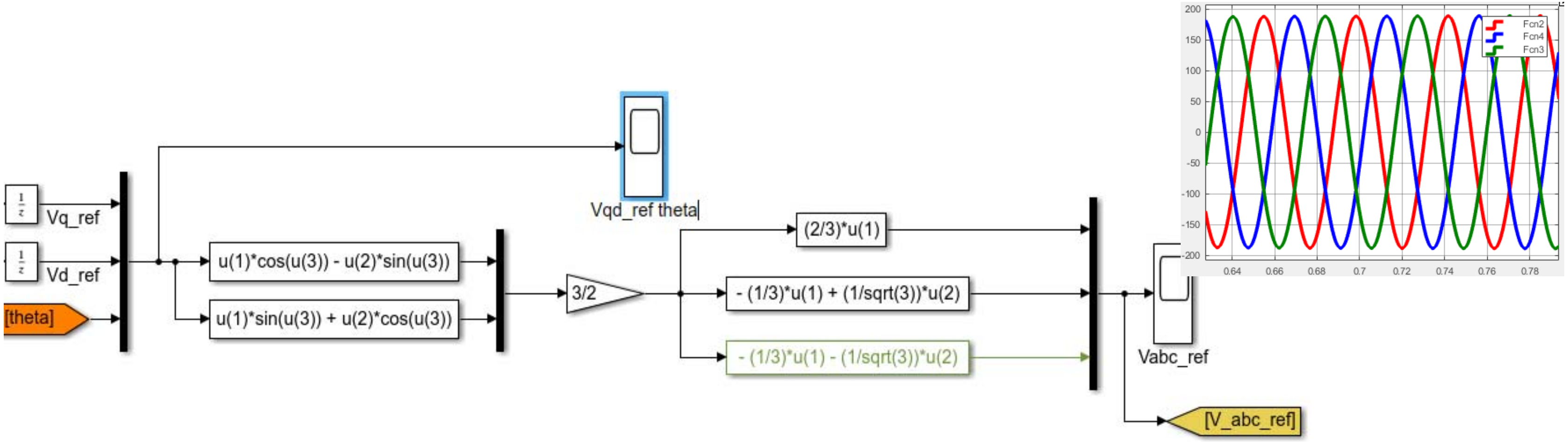


mechanical
back emf
torque
T_m
1/J1
1 / 0.2
K Ts
z-1
theta
omega
u+ [0 -2*pi/3 -4*pi/3]
cos
1.3
flux
flux_linkage
Ea
Eb
Ec
Va
Vb
Vc
T_e
Ia
Ib
Ic
I_abc

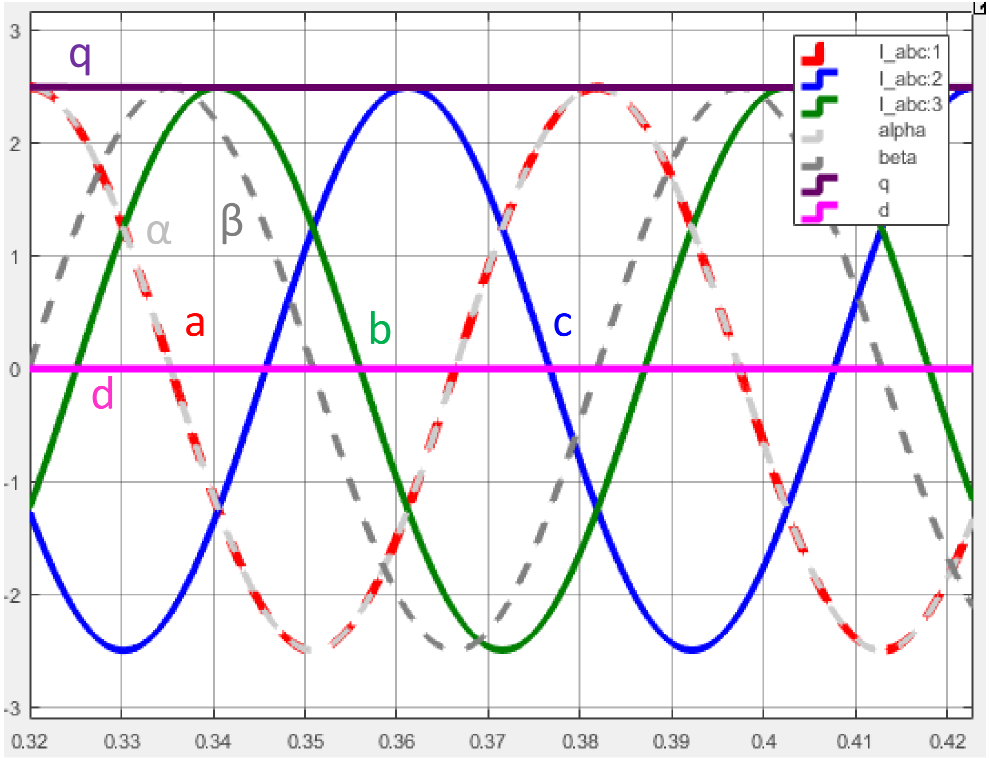
abc → dq and dq→abc







Steady State I_{abc}, $I_{\alpha\beta}$, I_{dq}



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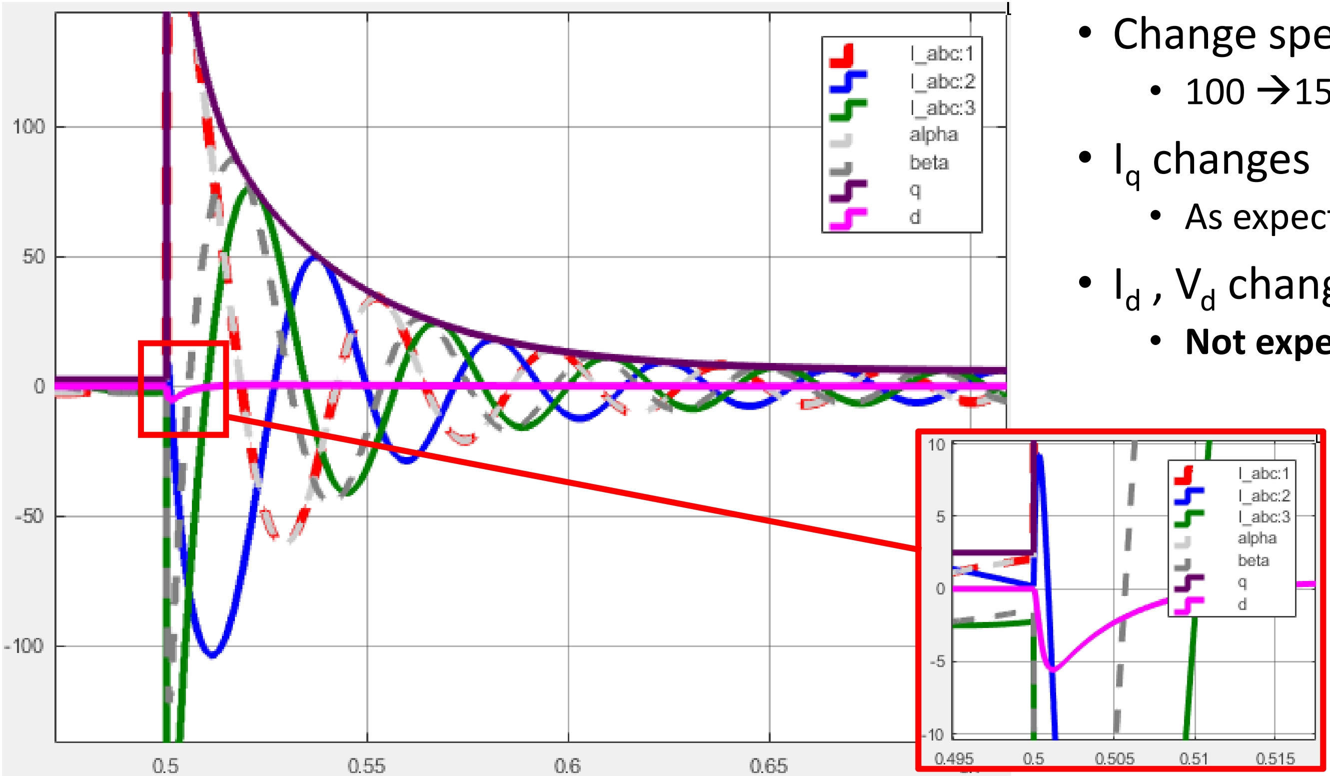
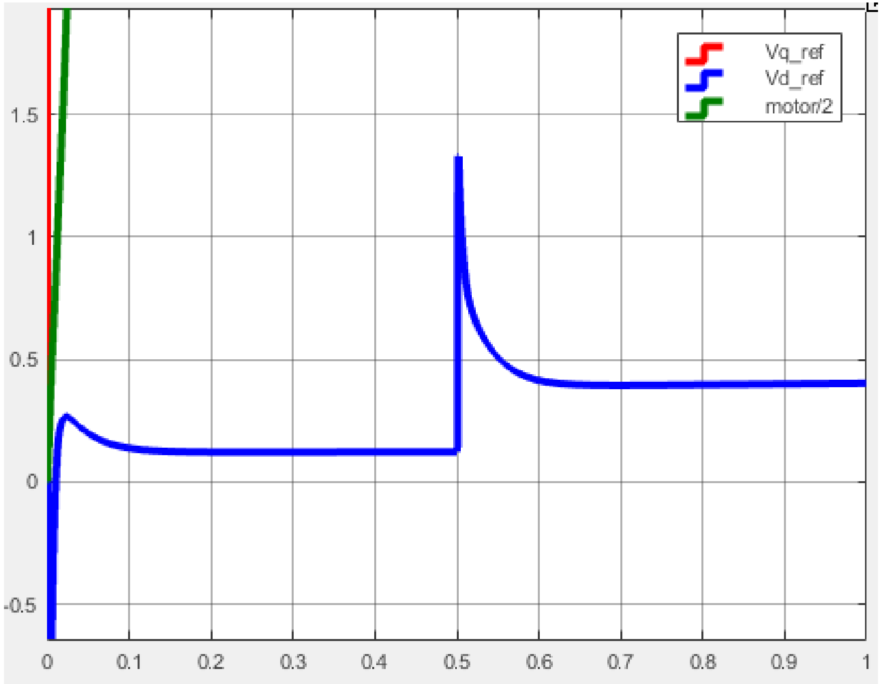
Transient I_{abc}, $I_{\alpha\beta}$, I_{dq}



- Change speed at t = 0.5sec
  - 100 →150
- I_q changes
  - As expected
- I_d , V_d change
  - **Not expected**

V_d Not Fixed

- I_d changes during transient
  - *dq* current control not as simple as it first appears .. *there goes our free lunch !*
- V_d not fixed
  - Changes with speed and current
  - Increase load by 10 (speed change at 0.5secs still occurs)



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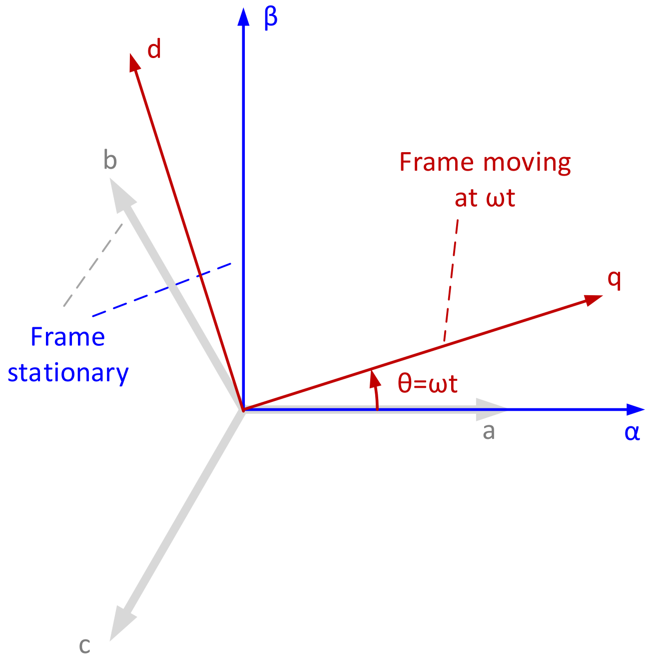
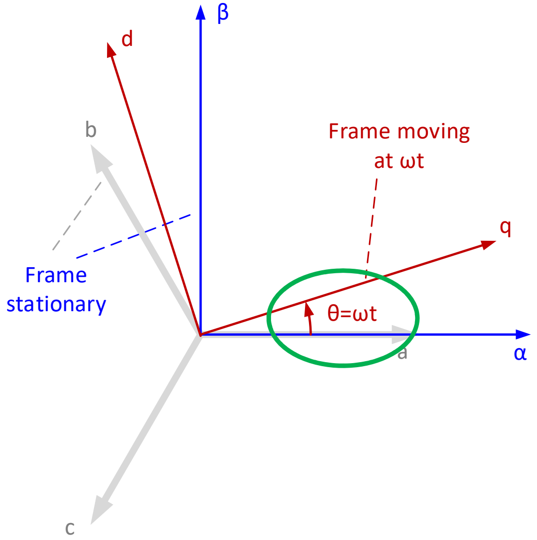
Motor Model in DQ (Rotor Frame) : Want v_{dq} in terms of λ_{dq}

- Saw this earlier :
  - $\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = T \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$
- In αβ frame, the motor equations are
  - $v_{\alpha\beta} = \frac{d}{dt}\lambda_{\alpha\beta}$ $\qquad v_{dq} \neq \frac{d}{dt}\lambda_{dq}$
    - λ : flux linkage
  - Ignored Resistance
    - Good approximation
  - Ignored leakage flux
    - For surface PM, not a good approximation
    - Do it here anyway, simplifies explanation
- $v_{dq} = T v_{\alpha\beta} = T \frac{d}{dt}\lambda_{\alpha\beta} = T \frac{d}{dt} T^{-1} \lambda_{dq}$



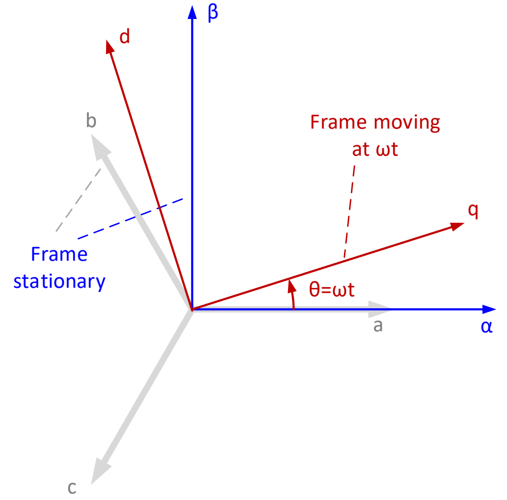
Motor Model in DQ

- $\begin{bmatrix} V_q \\ V_d \end{bmatrix} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix} = T \begin{bmatrix} V_\alpha \\ V_\beta \end{bmatrix}$
- $v_{qd} = T v_{\alpha\beta} = T \frac{d}{dt} \lambda_{\alpha\beta} = T \frac{d}{dt} T^{-1} \lambda_{qd}$
- $= \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \frac{d}{dt} \left\{ \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \lambda_{qd} \right\}$
- Recall that θ is a function of time, as is λ_{qd}



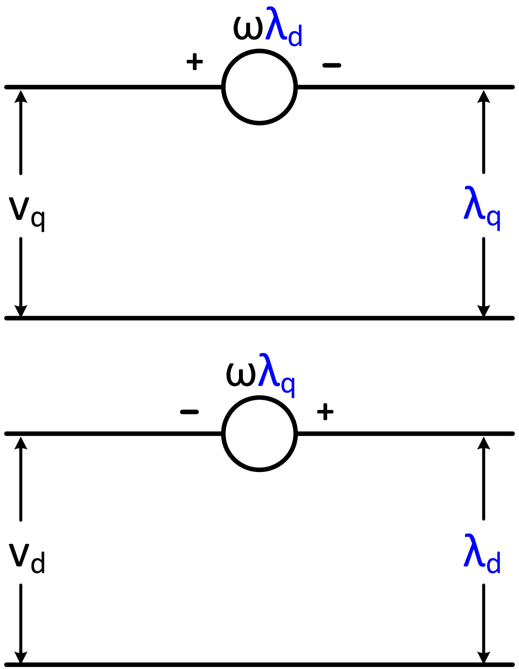
Motor Model in DQ

- Recall that
  - $\frac{d}{dt}(f(t) * g(t) = \dot{f} * g + gf * \dot{g}$
- $v_{qd} = \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \{\omega\lambda_{qd} \begin{bmatrix} -sin\theta & -cos\theta \\ cos\theta & -sin\theta \end{bmatrix} + \begin{bmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{bmatrix} \frac{d}{dt} \lambda_{qd}\}$
- $= \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \omega \begin{bmatrix} \lambda_q \\ \lambda_d \end{bmatrix} + \begin{bmatrix} \dot{\lambda}_q \\ \dot{\lambda}_d \end{bmatrix}$
- → *dq* axes cross-coupled



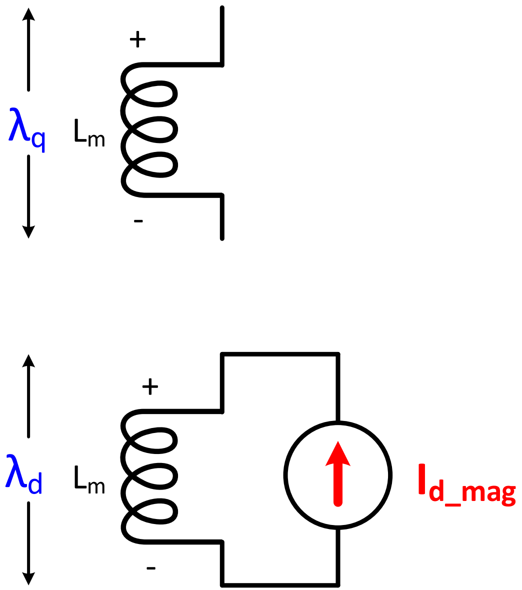
Stator Equivalent Circuit

- $v_{qd} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \omega \lambda_{qd}$



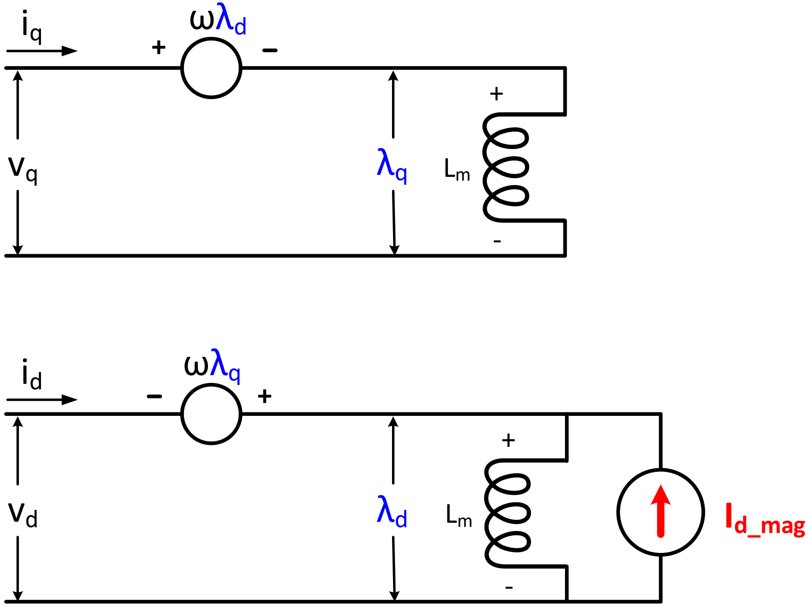
DQ Motor Equivalent Circuit (Rotor)

- Magnetic flux in *d* axis only
- Model permanent magnet as current source into an inductance
  - Called magnetizing inductance (L_m)
  - $L_m = L_d = L_q$
- L_m appears in *q* axis also, but no flux linkage from rotor



Equivalent Circuit of Complete Motor

- $\lambda_q = L_m * i_q$
- $\lambda_d = L_m * (I_{d\_mag} + i_d)$
- In steady state, $i_d = 0$
  - I_{d_mag} = constant
  - note : $\lambda_d >> \lambda_q$
- $v_q = \omega\lambda_d + L_m \frac{d}{dt} i_q$
- $v_d = -\omega\lambda_q + L_m \frac{d}{dt} i_d$
  - λ_d, λ_q as given above
- Details change somewhat when leakage is included
  - Does not change concepts involved



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Induction Motor

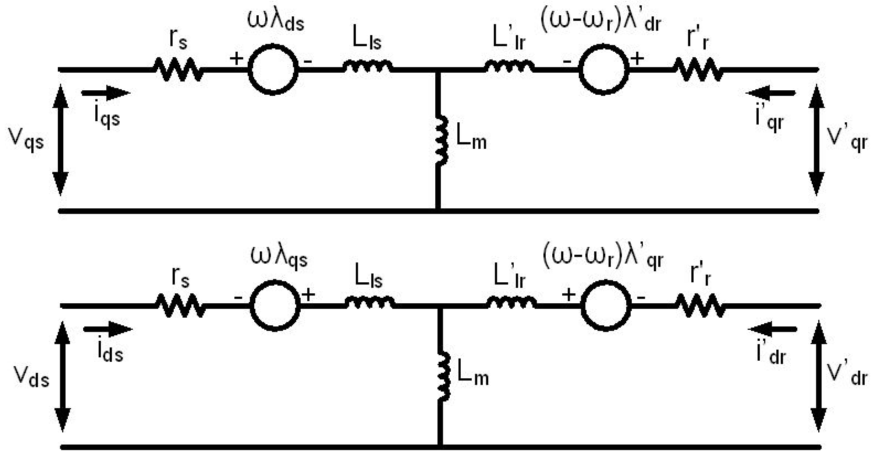
- Rotor field is induced
  - Rotor consists of shorted bars (i.e. ends of bars are shorted)
  - No magnet to produce it
- Stator must produce I_q (torque) and I_d(flux)
  - Still need I_q in phase with back emf
- I_d is not fixed, as it is for the PM
  - Dynamics are more complex
- PM doesn't have any rotor current
  - IM has to have I_{dr}, or there will be no flux
  - Adds complexity in equivalent circuit
- Same concepts of axes cross-coupling exists
- Need to know rotor position only for dq → abc and for abc → dq
  - Derive rotor speed from position
  - "Analysis of Electric Machinery", Krause .. Good reference

Induction Motor

- Induction motor airgap << PM airgap
  - Permeability of magnets close to unity
- Induction motor L_m >> PM L_m
  - Easy to field weaken IM
  - Tough to field weaken PM
- IM and PM are non-salient rotors
  - $L_d = L_q$
- IM has 4 current components in *qd* reference frame
  - Stator : I_{qs}, I_{ds}
  - Rotor : I_{qr}, I_{qr}
- PM dq transformation uses rotor frame
- IM dq transformation can be excitation frame or rotor frame
  - Excitation frame is synchronous ; rotor frame has slip ➔ is not synchronous

IM Equivalent Circuit

- Rotor reference frame
- Includes leakage components
- v_{qr}, v_{dr} = 0

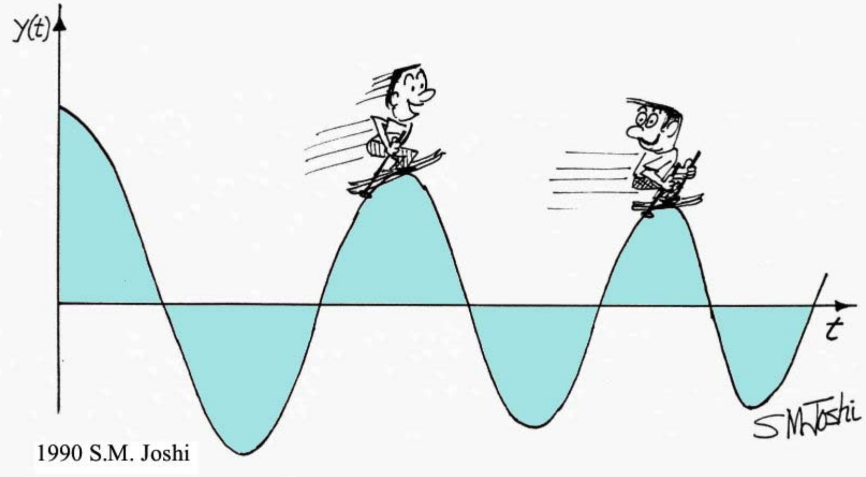


https://www.intechopen.com/books/induction-motors-modelling-and-control/modelling-and-analysis-of-squirrel-cage-induction-motor-with-leading-reactive-power-injection

Questions

- Thank you for attending

- *tony@pescinc.com*



"Don't you just love *underdamped systems?"*